Regularization and robust control of uncertain singular
discrete-time linear systems

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New sufficient linear matrix inequality (LMI) condition for regularization of singular discrete-time sys-
tems subject to norm-bounded uncertainties is given. Then a new class of feedback is proposed to stabilize
singular uncertain discrete-time systems with unknown time delays. The regularization and the stabiliz-
ability condition of this class of systems is given in terms of one strict LMI. A numerical example is
given to show the novelty of the control design.

Keywords: singular systems; linear matrix inequalities; regularization; discrete-time systems; system
theory.

1. Introduction

Recently, the problem of quadratic stability of uncertain singular discrete-time systems has drawn con-
siderable attention and numerous results on this topic have been reported in the literature (see, e.g. Xu &
Lam, 2004; Xu et al., 2001b; Dai, 1989). Moreover, quadratic stability and $H\infty$ control of both regular
and singular discrete systems with time delays have been also the subject of extensive research (see, e.g.
Mahmoud, 2000; Xu et al., 2001a,b; Lie & de Souza, 1997; Song et al., 1999; Kapila & Haddad, 1998;
Kim et al., 1996, and the references therein). However, the problem of regularization and robust stabil-
ization of singular discrete systems with unknown time delays has not been fully considered until now.
Singular systems also known as descriptor systems constitute a particular class of dynamical systems
which are subject to algebraic constraints. The stability and behavioural phenomenons of these systems
have been studied by many authors and their control is a challenging issue (Lewis, 1986; Dai, 1989;
Koumboulis & Mertzios, 1999; Yoshiyuki & Terra, 2002). We mean by ‘regularization’ the process of
making the singular system totally free from its algebraic constraints. To fulfil this objective, the action
of a predictive feedback is required. Unfortunately, the predictive feedback does not always exist and
the condition of existence of such feedback has not been studied until now, especially for discrete-time
systems subject to norm-bounded uncertainties. The majority of works that dealt with regularization
of singular systems were essentially developed in the continuous time case. Moreover, none of these
methods has given a systematic procedure to compute, in efficient way, the gains of such regularizing
feedbacks (see, e.g. Wang & Soh, 1999; Özçaldiran & Lewis, 1990; Chu & Cai, 2000; Chu et al., 1999;
Bunce-Gerstner et al., 1999, and the references therein).

The objective of this paper is twofolds. First, we begin by exposing sufficient linear matrix in-
equality (LMI) for regularization of singular discrete-time systems subject to norm-bounded uncertainties.
Second, we develop a combined memoryless and predictive state feedback that realizes both the reg-
ularization and the asymptotic stabilization of the time delay discrete-time singular systems having
uncertainties in all the nominal matrices. The condition of existence of such regularizing stabilizing controllers is formulated in terms of one strict LMI. The developed sufficient conditions for regularization and stabilization are delay independent which makes the resulting controller valid for a wide range of time delays. This result can be seen as an extension to the numerous results on robust stability of uncertain discrete-time delay systems that were developed in terms of non-linear matrix inequalities (see, e.g. Xu et al., 2001a; Mahmoud, 2000, and the references therein). Finally, illustrative example is provided to demonstrate the applicability of the proposed method.

The rest of the paper is as follows: In Section 2, a convex optimization approach to regularization of discrete-time singular systems with norm-bounded uncertainties is exposed. Robust stabilization and regularization of the closed-loop system will be analysed in Section 3. To show the effectiveness of the proposed method, in Section 4 we give a numerical example. Finally, we end by some concluding remarks.

The notation $A > 0$ (respectively, $A < 0$) means that the matrix $A$ is positive definite (respectively, negative definite). We denote by $A'$ the matrix transpose of $A$. We note by $I$ and $0$ the identity matrix and the null matrix of appropriate dimensions, respectively. $\mathbb{Z}_{\geq 0}$ and $\mathbb{R}$ stands for the sets of positive integer numbers and real numbers, respectively. The Schur complement lemma is frequently used in setting the proofs of statements. For this reason we prefer to recall this result.

**Lemma 1 (The Schur Complement Lemma)** For given constant matrices $M, N, Q$ of appropriate dimensions where $M$ and $Q$ are symmetric, $Q > 0$ and $M + N'Q^{-1}N < 0$ if and only if

$$
\begin{bmatrix}
M & N' \\
N & -Q
\end{bmatrix} < 0,
$$

or equivalently

$$
\begin{bmatrix}
-Q & N \\
N' & M
\end{bmatrix} < 0.
$$

**Proof.** For the proof, see Boyd et al. (1994). □

Before tackling the main problems of this paper, we should also recall the following fact.

**Fact 1** For given matrices $\Sigma_1$ and $\Sigma_2$ with appropriate dimensions, we have

$$
\Sigma_1' \Sigma_2 + \Sigma_2' \Sigma_1 \leq \mu \Sigma_1' \Sigma_1 + \mu^{-1} \Sigma_2' \Sigma_2
$$

and

$$
\Sigma_1' \Sigma_2 + \Sigma_2' \Sigma_1 \leq \Sigma_1' \Sigma_1 + \Sigma_2' \Sigma_2,
$$

where $\mu$ is any positive constant and $P$ is an arbitrary symmetric positive definite matrix of appropriate dimension.

### 2. Regularization

Consider the discrete-time singular system with time delay and subject to norm-bounded uncertainties

$$
Ex_{k+1} = (A + \Delta A)x_k + (A_d + \Delta A_d)x_{k-d} + (B + \Delta B)u_k,
$$

(1)
where \( x_k \in \mathbb{R}^n \) is the state vector and \( u_k \in \mathbb{R}^m \) is the input vector. \( E \) is a singular real matrix and \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, A_d \in \mathbb{R}^{n \times n} \) are constant real matrices. \( d \) is the unknown amount of the system delay. We assume that the system uncertainties have the following standard structures:

\[
\Delta A = M_A F_A(k) N_A \in \mathbb{R}^{n \times n},
\]

\[
\Delta B = M_B F_B(k) N_B \in \mathbb{R}^{n \times m},
\]

\[
\Delta A_d = M_D F_D(k) N_D \in \mathbb{R}^{n \times n},
\]

where \( M_A \in \mathbb{R}^{n \times n}, M_D \in \mathbb{R}^{n \times n}, M_B \in \mathbb{R}^{n \times m}, N_A \in \mathbb{R}^{n \times n}, N_D \in \mathbb{R}^{n \times n} \) and \( N_B \in \mathbb{R}^{m \times m} \) are constant known matrices and \( F_A(k), F_B(k), F_D(k) \) are time-dependent unknown matrices that verify

\[
F_A'(k) F_A(k) \leq I, \quad \forall k \in \mathbb{Z}_{\geq 0},
\]

\[
F_B'(k) F_B(k) \leq I, \quad \forall k \in \mathbb{Z}_{\geq 0},
\]

\[
F_D'(k) F_D(k) \leq I, \quad \forall k \in \mathbb{Z}_{\geq 0}.
\]

**Definition 1** System (1) is regularized by the feedback

\[
u_k = L x_{k+1} + w_k,
\]

if there exist a gain \( L \in \mathbb{R}^{m \times n} \) such that System (1) under the feedback (3) is equivalent to the system

\[
E_r x_{k+1} = (A + \Delta A) x_k + (A_d + \Delta A_d) x_{k-d} + (B + \Delta B) w_k,
\]

where \( E_r \) is a full rank matrix and \( w_k \in \mathbb{R}^m \) is the new control input.

Remark that if the uncertainties \( \Delta A, \Delta B, \Delta A_d \) are absent, then the dynamics of System (4) can be written as

\[
x_{k+1} = E_r^{-1} A x_k + E_r^{-1} A_d x_{k-d} + E_r^{-1} B w_k,
\]

which means that all the properties of singular systems will disappear under the action of the regularizing feedback (3). If the uncertainties are present, rewriting System (4) in terms of separate known and unknown nominal matrices is not possible, but this will not prevent the search of a stabilizing controller as it will be shown in Section 3.

The regularization of System (1) by the feedback (3) turns on checking the invertibility of the matrix \( E - (B + \Delta B)L \) for all possible uncertainties \( \Delta B \). This problem has not been considered until now and the method of seeking \( L \) constitutes the major difficulty since \( \Delta B \) is not completely known. For this purpose, we will give sufficient LMI condition for the existence and computation of such gain.

The main result of this section is given in the following statement.

**Theorem 1** System (1) is regularized by the feedback

\[
u_k = Y P^{-1} x_{k+1} + w_k
\]

or the matrix \( E - (B + M_B F_B(k) N_B) Y P^{-1} \) has a full rank if there exist a positive and definite matrix \( P \in \mathbb{R}^{n \times n} \), a matrix \( Y \in \mathbb{R}^{m \times n} \) and positive constant \( \epsilon_y \) such that the following LMI holds:

\[
\begin{bmatrix}
    P E' + E P - B Y - Y' B' - \epsilon_y M_B M_B' & Y' N_B' & P \\
    N_B Y & \epsilon_y I & 0 \\
    P & 0 & P
\end{bmatrix} > 0.
\]

(6)
Proof. Let $B_\Delta = B + \Delta B$, then the matrix $E - B_\Delta Y P^{-1}$ has a full rank if

$$(E - B_\Delta Y P^{-1})' P^{-1} (E - B_\Delta Y P^{-1}) > 0. \quad (7)$$

The last inequality can be rewritten as follows:

$$(E' P^{-1} - P^{-1} Y' B'_\Delta P^{-1}) P (P^{-1} E - P^{-1} B_\Delta Y P^{-1}) > 0. \quad (8)$$

If we put $X = P^{-1} E - P^{-1} B_\Delta Y P^{-1}$ and $Z = I$, then using the result of Fact 1, we can write

$$X' P X + Z' P^{-1} Z \geq X' Z + Z' X, \quad (9)$$

or

$$(E' P^{-1} - P^{-1} Y' B'_\Delta P^{-1}) P (P^{-1} E - P^{-1} B_\Delta Y P^{-1}) \geq E' P^{-1} - P^{-1} Y' B'_\Delta P^{-1} + P^{-1} E - P^{-1} B_\Delta Y P^{-1} - P^{-1}. \quad (10)$$

If

$$E' P^{-1} - P^{-1} Y' B'_\Delta P^{-1} + P^{-1} E - P^{-1} B_\Delta Y P^{-1} - P^{-1} > 0,$$

then inequality (7) is satisfied, and hence the matrix $E - (B + M_B F_B(k) N_B) Y P^{-1}$ has a full rank. Pre- and post-multiplying inequality (10) by the matrix $P$, we obtain

$$P E' + E P - B_\Delta Y - Y' B'_\Delta - P > 0. \quad (11)$$

From the last inequality and using the result of Fact 1, we can write for some $\epsilon_b > 0$,

$$P E' + E P - B_\Delta Y - Y' B'_\Delta - P > P E' + E P - B Y - Y' B' - \epsilon_b M_B M'_B - \epsilon_b^{-1} Y' N'_B N_B Y - P.$$

By the Schur complement lemma, the matrix

$$P E' + E P - B Y - Y' B' - \epsilon_b M_B M'_B - \epsilon_b^{-1} Y' N'_B N_B Y - P$$

is positive definite if the following LMI holds:

$$
\begin{bmatrix}
PE' + EP - BY - Y'B' - \epsilon_b M_B M'_B - \epsilon_b^{-1} Y' N'_B N_B Y - P & Y' N'_B \\
N_B Y & \epsilon_b I
\end{bmatrix} > 0.
$$

By the Schur complement, the last LMI is equivalent to (6). This ends the proof. \qed

3. Stabilizability

The aim of this section is to deal with the regularization and the quadratic stabilization of System (1) for all admissible uncertainties $\Delta A$, $\Delta B$ and $\Delta A_d$. The objective is to make the closed-loop system stable and non-singular under the action of a feedback of the form $u_k = L x_{k+1} + K x_k$, where $L$ and $K$ are
constant matrices to be determined. By the application of such feedback, the resulting system behaves as a stable system of the form
\[ x_{k+1} = \mathcal{A}_c x_k + \mathcal{A}_d x_{k-d}, \]  
(12)
where \( \mathcal{A}_c \) and \( \mathcal{A}_d \) are the closed-loop \( n \times n \) dimensional matrices that are not necessarily known. By the action of the feedback part \( L x_{k+1} \), System (1) liberates from various phenomenons of singular systems and hence, a multi-objective controller design can be easily done by appropriate choice of the gain \( K \). The design of the feedback gains is given by the following theorem.

**Theorem 2** Consider System (1). If there exist a positive and definite matrix \( P \in \mathbb{R}^{n \times n} \), two matrices \( Y_1 \in \mathbb{R}^{m \times n} \) and \( Y_2 \in \mathbb{R}^{m \times n} \), and a set of positive constants \( \epsilon_a, \epsilon_b, \bar{\epsilon}_b \) and \( \epsilon_d \) such that the following LMI holds:
\[
\begin{bmatrix}
\mathcal{M}_{1,1} & 0 & P A' + Y_2' B' & 0 & P & P N_A' & Y_2' N_B' & Y_1' N_B' \\
0 & -Q & Q A_d' & Q N_d' & 0 & 0 & 0 & 0 \\
A P + B Y_2 & A_d Q & \mathcal{M}_{3,3} & 0 & 0 & 0 & 0 & 0 \\
0 & N_D Q & 0 & -\epsilon_d I & 0 & 0 & 0 & 0 \\
P & 0 & 0 & 0 & -Q & 0 & 0 & 0 \\
N_A P & 0 & 0 & 0 & 0 & -\epsilon_a I & 0 & 0 \\
N_B Y_2 & 0 & 0 & 0 & 0 & 0 & -\epsilon_b I & 0 \\
N_B Y_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\epsilon}_b I \\
\end{bmatrix} \prec 0,
\]
(13)
where \( \mathcal{M}_{1,1} = -P E' - E P + B Y_1 + Y_2' B' + \bar{\epsilon}_b M_B M_B' + P \), \( \mathcal{M}_{3,3} = -P + \epsilon_a M_A M_A' + \epsilon_d M_D M_D' + \epsilon_b M_B M_B' \). Then System (1) is asymptotically stable under the feedback
\[ u_k = Y_1 P^{-1} x_{k+1} + Y_2 P^{-1} x_k, \]  
(14)
and the matrix \( E - (B + M_B F_B (k) N_B) Y P^{-1} \) has a full rank.

**Proof.** For notation simplicity, let \( A_{\Delta} = A + \Delta A \), \( B_{\Delta} = B + \Delta B \), \( A_{d\Delta} = A_d + \Delta A_d \), \( A_{c\Delta} = A_c + B_{\Delta} Y_2 P^{-1} \), \( E_{\Delta} = E - B_{\Delta} Y_1 P^{-1} \). Under the action of controller (14), the dynamics of the closed-loop system is governed by
\[ E_{\Delta} x_{k+1} = A_{c\Delta} x_k + A_{d\Delta} x_{k-d}. \]  
(15)
Taking the Lyapunov function candidate
\[ V_k = x_k' E_{\Delta} P^{-1} E_{\Delta} x_k + \sum_{i=k-d}^{k-1} x_i' Q^{-1} x_i, \]  
(16)
we have
\[
V_{k+1} - V_k = x_{k+1}' E_{\Delta} P^{-1} E_{\Delta} x_{k+1} - x_k' E_{\Delta} P^{-1} E_{\Delta} x_k + x_k' Q^{-1} x_k - x_{k-d}' Q^{-1} x_{k-d} \\
= x_k' (Q^{-1} - E_{\Delta}' P^{-1} E_{\Delta} + A_{c\Delta}' P^{-1} A_{c\Delta}) x_k + x_k' A_{c\Delta}' P^{-1} A_{d\Delta} x_{k-d} \\
+ x_{k-d}' A_{d\Delta}' P^{-1} A_{c\Delta} x_k + x_{k-d}' A_{d\Delta}' P^{-1} A_{d\Delta} x_{k-d} - x_{k-d}' Q^{-1} x_{k-d}.
\]
The last difference $V_{k+1} - V_k$ can be rewritten as
\[
\begin{bmatrix}
x_k \\
x_{k-d}
\end{bmatrix}'
\begin{bmatrix}
A_{1,1} & A_{1,2} \\
A_{2,1} & A_{2,2}
\end{bmatrix}
\begin{bmatrix}
x_k \\
x_{k-d}
\end{bmatrix},
\]  
where
\[
A_{1,1} = Q^{-1} - E^\prime_{r\Delta} P^{-1} E_{r\Delta} + A^\prime_{c\Delta} P^{-1} A_{c\Delta},
\]
\[
A_{1,2} = A^\prime_{c\Delta} P^{-1} A_{d\Delta},
\]
\[
A_{2,1} = A^\prime_{1,2},
\]
\[
A_{2,2} = -Q^{-1} + A^\prime_{d\Delta} P^{-1} A_{d\Delta}.
\]

System (1) is stable under the action of the feedback (14) if
\[
\begin{bmatrix}
A_{1,1} & A_{1,2} \\
A_{2,1} & A_{2,2}
\end{bmatrix}
< 0.
\]  
(18)

The last inequality is equivalent to the following LMI:
\[
\begin{bmatrix}
-P E^\prime_{r\Delta} P^{-1} E_{r\Delta} P + P Q^{-1} P & 0 & P A^\prime_{c\Delta} \\
0 & -Q & Q A^\prime_{d\Delta} \\
A_{c\Delta} P & A_{d\Delta} Q & -P
\end{bmatrix}
< 0.
\]  
(19)

It is easy to verify that inequality (18) can be re-obtained by pre- and post-multiplying inequality (19) by the matrix
\[
\begin{bmatrix}
P^{-1} & 0 & A^\prime_{c\Delta} P^{-1} \\
0 & Q^{-1} & A^\prime_{d\Delta} P^{-1}
\end{bmatrix}.
\]  
(20)

We rewrite inequality (19) as
\[
\begin{bmatrix}
-P E^\prime_{r\Delta} P^{-1} E_{r\Delta} P + P Q^{-1} P & 0 & P A^\prime + Y^\prime_2 B' \\
0 & -Q & Q A'_{d} \\
A P + B Y_2 & A_d Q & -P
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & P \Delta A'
\\
0 & 0 & 0 \\
\Delta A P & 0 & 0
\end{bmatrix}
\]
\[
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & Q \Delta A'_d \\
0 & \Delta A_d Q & 0
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & Y^\prime_2 \Delta B'
\\
0 & 0 & 0 \\
\Delta B Y_2 & 0 & 0
\end{bmatrix}
< 0.
\]

Using the definitions of the uncertainties and the result of Fact 1, we have
\[
\begin{bmatrix}
0 & 0 & P \Delta A' \\
0 & 0 & 0 \\
\Delta A P & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
M_A
\end{bmatrix} F_A(k)[N_A P \ 0 \ 0] + [N_A P \ 0 \ 0] F_A^\prime(k) \begin{bmatrix}
0 \\
0 \\
M_A
\end{bmatrix}.
\]
Using the result of Fact 1 with $F_A'(k)F_A(k) \leq I$, we can write that for any $\epsilon_a > 0$

$$
\begin{bmatrix}
0 & 0 & P \Delta A'
\Delta AP & 0 & 0
\end{bmatrix} \leq \epsilon_a^{-1}
\begin{bmatrix}
PN_A'
0 & 0 & [N_A P & 0 & 0]
0 & 0 & M_A'
\end{bmatrix}
= 
\begin{bmatrix}
\epsilon_a^{-1}PN_AN_A P & 0 & 0
0 & 0 & 0
0 & 0 & \epsilon_a M_AM_A'
\end{bmatrix}.
$$

Similarly,

$$
\begin{bmatrix}
0 & 0 & 0
0 & Q \Delta A_d'
\Delta A_d Q & 0
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0
0 & 0 & 0
\end{bmatrix}
F_D(k)[0 & N_d Q & 0] + [0 & N_d Q & 0]'F_D'(k)
\begin{bmatrix}
0
\end{bmatrix}'.

By the use of Fact 1 and taking into account $F_D'(k)F_D(k) \leq I$, then according to the last equality, we can deduce that for any $\epsilon_d > 0$

$$
\begin{bmatrix}
0 & 0 & 0
0 & 0 & Q \Delta A_d'
\Delta A_d Q & 0
\end{bmatrix} \leq \epsilon_d^{-1}
\begin{bmatrix}
QN_D'
0 & 0 & [N_D Q & 0]
0 & 0 & M_D'
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 0
0 & \epsilon_d^{-1}QN_DN_D Q & 0
0 & 0 & \epsilon_d M_DM_D'
\end{bmatrix}.
$$

With the same analysis, we can write that for all $F_B'(k)F_B(k) \leq I$ and for all $\epsilon_b > 0$

$$
\begin{bmatrix}
0 & 0 & Y_2 \Delta B'
\Delta B Y_2 & 0 & 0
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0
0 & 0 & 0
\end{bmatrix}
F_B(k)[N_B Y_2 & 0 & 0] + [N_B Y_2 & 0 & 0]'F_B'(k)
\begin{bmatrix}
0
\end{bmatrix}'.
$$

\begin{align*}
&\leq \epsilon_b^{-1}Y_2'N_B'N_B Y_2 & 0 & 0
&\quad 0 & 0 & 0
&\quad 0 & 0 & \epsilon_b M_B M_B'.
\end{align*}

From the last development, we obtain a sufficient condition to fulfill inequality (19), i.e.

$$
\begin{bmatrix}
\mathcal{H}_{1,1} & 0 & PA' + Y_2'B'
0 & \mathcal{H}_{2,2} & Q A_d'
AP + B Y_2 & A_d Q & \mathcal{H}_{3,3}
\end{bmatrix} < 0,
$$

(21)
where
\[ \mathcal{H}_{1,1} = -PE_{r_\Delta} P^{-1} E_{r_\Delta} P + P Q^{-1} P + \epsilon_a^{-1} P N_A' N_A P + \epsilon_b^{-1} Y_2' N_B' N_B Y_2, \]
\[ \mathcal{H}_{2,2} = -Q + \epsilon_a^{-1} Q N_D' N_D Q, \]
\[ \mathcal{H}_{3,3} = -P + \epsilon_a M_A M_A' + \epsilon_d M_D M_D' + \epsilon_b M_B M_B'. \]

Applying the Schur complement lemma, (21) is equivalent to
\[
\begin{bmatrix}
\mathcal{H}_{1,1} & 0 & PA' + Y_2' B' & 0 \\
0 & -Q & QA_d' & QN_D' \\
AP + BY_2 & A_d Q & \mathcal{H}_{3,3} & 0 \\
0 & N_D Q & 0 & -\epsilon_d I
\end{bmatrix} < 0. \tag{22}
\]

The last inequality is also equivalent by the Schur complement to the following inequality:
\[
\begin{bmatrix}
-PE_{r_\Delta} P^{-1} E_{r_\Delta} P & 0 & PA' + Y_2' B' & 0 & P & PN_A' & Y_2' N_B' \\
0 & -Q & QA_d' & QN_D' & 0 & 0 & 0 \\
AP + BY_2 & A_d Q & \mathcal{H}_{3,3} & 0 & 0 & 0 & 0 \\
0 & N_D Q & 0 & -\epsilon_d I & 0 & 0 & 0 \\
P & 0 & 0 & 0 & -Q & 0 & 0 \\
N_A P & 0 & 0 & 0 & 0 & -\epsilon_a I & 0 \\
N_B Y_2 & 0 & 0 & 0 & 0 & 0 & -\epsilon_b I
\end{bmatrix} < 0. \tag{23}
\]

From the result of Theorem 1, we conclude that for \( \tilde{\epsilon}_b > 0, \)
\[
-PE_{r_\Delta} P^{-1} E_{r_\Delta} P < -PE' - EP + BY_1 + Y_1' B' + \tilde{\epsilon}_b M_B M_B' + \tilde{\epsilon}_b^{-1} Y_1' N_B' N_B Y_1 + P. \tag{24}
\]

From inequality (23) and using (24), the condition of stability of System (1) is equivalent to (13). This ends the proof.

\[ \square \]

**Remark 1** According to (23), we remark that the negativity of the element \(-PE_{r_\Delta} P^{-1} E_{r_\Delta} P\) is a necessary condition. In the meantime, the condition \(-PE_{r_\Delta} P^{-1} E_{r_\Delta} P < 0\) is exactly the condition of the regularization of System (1), see inequality (8).

The LMI (13) gives a sufficient condition for regularization and stabilization of System (1) by means of combined memoryless and predictive feedback. Moreover, the LMI (13) is delay independent which makes the design of feedback gains, if they exist, valid for arbitrary amount of delay.
4. Illustrative example

To show the effectiveness of the developed results, consider the singular discrete-time delay system with the following nominal matrices:

\[
E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & 5 & 1 \\ 3 & -2 & -1 \\ -1 & -1 & 1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.1 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.3 \\ -0.1 & -0.5 & 0.3 \end{bmatrix},
\]

\[
B = \begin{bmatrix} 4 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad M_A = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.2 & 0.1 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{bmatrix},
\]

\[
M_B = \begin{bmatrix} 2 & 1 \\ 1 & 0.9 \\ 2 & 1 \end{bmatrix}, \quad M_D = \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.02 & 0.01 & 0.3 \\ 0.1 & 0.1 & 0.3 \end{bmatrix},
\]

\[
N_A = I, \quad N_B = 0.1I, \quad N_D = 0.1I.
\]

To solve LMI (13), we have used the LMI package of Matlab which gives us the following solution

\[
\epsilon_a = 0.2974, \quad \epsilon_b = 0.0293, \quad \tilde{\epsilon}_b = 0.0289, \quad \epsilon_d = 0.0402
\]

and

\[
Q = \begin{bmatrix} 1.1757 & -0.2594 & -0.0421 \\ -0.2594 & 0.1345 & 0.0084 \\ -0.0421 & 0.0084 & 0.1233 \end{bmatrix},
\]

\[
P = \begin{bmatrix} 0.5048 & 0.3193 & 0.4634 \\ 0.3193 & 0.2315 & 0.3114 \\ 0.4634 & 0.3114 & 0.5205 \end{bmatrix},
\]

\[
Y_1 = \begin{bmatrix} -2.0931 & -0.0117 & 0.6577 \\ 0.6055 & -1.1859 & -3.0287 \end{bmatrix},
\]

\[
Y_2 = \begin{bmatrix} -0.7771 & -0.6553 & -0.9839 \\ 1.1860 & 0.3696 & 0.2549 \end{bmatrix}.
\]

5. Conclusion

In this paper, a new LMI condition for regularization of singular discrete systems with time delay and norm-bounded uncertainties is given. We showed that the action of regularizing feedback has permitted us to formulate the stabilizability condition of the singular system in terms of strict LMI. The benefits
of adding a regularizing feedback is crucial to liberate from a lot of transient phenomenons of singular systems and hence, more optimality conditions on the design of the memoryless stabilizing feedback can be imposed. Observer-based control and regularization of this class of systems is under investigation and shall be the topic of our next contribution.

REFERENCES


