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Regularization and robust control of uncertain singular discrete-time linear systems

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New sufficient linear matrix inequality (LMI) condition for regularization of singular discrete-time systems subject to norm-bounded uncertainties is given. Then a new class of feedback is proposed to stabilize singular uncertain discrete-time systems with unknown time delays. The regularization and the stabilizability condition of this class of systems is given in terms of one strict LMI. A numerical example is given to show the novelty of the control design.

Keywords: singular systems; linear matrix inequalities; regularization; discrete-time systems; system theory.

1. Introduction

Recently, the problem of quadratic stability of uncertain singular discrete-time systems has drawn considerable attention and numerous results on this topic have been reported in the literature (see, e.g. Xu & Lam, 2004; Xu *et al.*, 2001b; Dai, 1989). Moreover, quadratic stability and H_∞ control of both regular and singular discrete systems with time delays have been also the subject of extensive research (see, e.g. Mahmoud, 2000; Xu *et al.*, 2001a,b; Lie & de Souza, 1997; Song *et al.*, 1999; Kapila & Haddad, 1998; Kim *et al.*, 1996, and the references therein). However, the problem of regularization and robust stabilization of singular discrete systems with unknown time delays has not been fully considered until now. Singular systems also known as descriptor systems constitute a particular class of dynamical systems which are subject to algebraic constraints. The stability and behavioural phenomenons of these systems have been studied by many authors and their control is a challenging issue (Lewis, 1986; Dai, 1989; Koumboulis & Mertzios, 1999; Yoshiyuki & Terra, 2002). We mean by 'regularization' the process of making the singular system totally free from its algebraic constraints. To fulfil this objective, the action of a predictive feedback is required. Unfortunately, the predictive feedback does not always exist and the condition of existence of such feedback has not been studied until now, especially for discrete-time systems subject to norm-bounded uncertainties. The majority of works that dealt with regularization of singular systems were essentially developed in the continuous time case. Moreover, none of these methods has given a systematic procedure to compute, in efficient way, the gains of such regularizing feedbacks (see, e.g. Wang & Soh, 1999; Özçaldiran & Lewis, 1990; Chu & Cai, 2000; Chu *et al.*, 1999; Bunce-Gerstner *et al.*, 1999, and the references therein).

The objective of this paper is twofolds. First, we begin by exposing sufficient linear matrix inequality (LMI) for regularization of singular discrete-time systems subject to norm-bounded uncertainties. Second, we develop a combined memoryless and predictive state feedback that realizes both the regularization and the asymptotic stabilization of the time delay discrete-time singular systems having

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uncertainties in all the nominal matrices. The condition of existence of such regularizing stabilizing controllers is formulated in terms of one strict LMI. The developed sufficient conditions for regularization and stabilization are delay independent which makes the resulting controller valid for a wide range of time delays. This result can be seen as an extension to the numerous results on robust stability of uncertain discrete-time delay systems that were developed in terms of non-linear matrix inequalities (see, e.g. Xu *et al.*, 2001a; Mahmoud, 2000, and the references therein). Finally, illustrative example is provided to demonstrate the applicability of the proposed method.

The rest of the paper is as follows: In Section 2, a convex optimization approach to regularization of discrete-time singular systems with norm-bounded uncertainties is exposed. Robust stabilization and regularization of the closed-loop system will be analysed in Section 3. To show the effectiveness of the proposed method, in Section 4 we give a numerical example. Finally, we end by some concluding remarks.

The notation $A > 0$ (respectively, $A < 0$) means that the matrix A is positive definite (respectively, negative definite). We denote by A' the matrix transpose of A. We note by I and $\mathbf{0}$ the identity matrix and the null matrix of appropriate dimensions, respectively. $\mathbb{Z}_{\geqslant 0}$ and $\mathbb{R}$ stands for the sets of positive integer numbers and real numbers, respectively. The Schur complement lemma is frequently used in setting the proofs of statements. For this reason we prefer to recall this result.

LEMMA 1 (THE SCHUR COMPLEMENT LEMMA) For given constant matrices M, N, Q of appropriate dimensions where M and Q are symmetric, $Q > 0$ and $M + N'Q^{-1}N < 0$ if and only if

$$\begin{bmatrix} M & N' \\ N & -Q \end{bmatrix} < 0,$$

or equivalently

$$\begin{bmatrix} -Q & N \\ N' & M \end{bmatrix} < 0.$$

Proof. For the proof, see Boyd *et al.* (1994). □

Before tackling the main problems of this paper, we should also recall the following fact.

FACT 1 For given matrices Σ_1 and Σ_2 with appropriate dimensions, we have

$$\Sigma_1'\Sigma_2 + \Sigma_2'\Sigma_1 \leqslant \mu\Sigma_1'\Sigma_1 + \mu^{-1}\Sigma_2'\Sigma_2$$

and

$$\Sigma_1'\Sigma_2 + \Sigma_2'\Sigma_1 \leqslant \Sigma_1'P^{-1}\Sigma_1 + \Sigma_2'P\Sigma_2,$$

where μ is any positive constant and P is an arbitrary symmetric positive definite matrix of appropriate dimension.

2. Regularization

Consider the discrete-time singular system with time delay and subject to norm-bounded uncertainties

$$Ex_{k+1} = (A + \Delta A)x_k + (A_d + \Delta A_d)x_{k-d} + (B + \Delta B)u_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector and $u_k \in \mathbb{R}^m$ is the input vector. E is a singular real matrix and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $A_d \in \mathbb{R}^{n\times n}$ are constant real matrices. d is the unknown amount of the system delay. We assume that the system uncertainties have the following standard structures:

$$\begin{aligned}\Delta A &= M_A F_A(k) N_A \in \mathbb{R}^{n\times n},\\ \Delta B &= M_B F_B(k) N_B \in \mathbb{R}^{n\times m},\\ \Delta A_d &= M_D F_D(k) N_D \in \mathbb{R}^{n\times n},\end{aligned} \tag{2}$$

where $M_A \in \mathbb{R}^{n\times n}$, $M_D \in \mathbb{R}^{n\times n}$, $M_B \in \mathbb{R}^{n\times m}$, $N_A \in \mathbb{R}^{n\times n}$, $N_D \in \mathbb{R}^{n\times n}$ and $N_B \in \mathbb{R}^{m\times m}$ are constant known matrices and $F_A(k)$, $F_B(k)$, $F_D(k)$ are time-dependent unknown matrices that verify

$$\begin{aligned}F_A'(k)F_A(k) &\leqslant I, \quad \forall k \in \mathbb{Z}_{\geqslant 0},\\ F_B'(k)F_B(k) &\leqslant I, \quad \forall k \in \mathbb{Z}_{\geqslant 0},\\ F_D'(k)F_D(k) &\leqslant I, \quad \forall k \in \mathbb{Z}_{\geqslant 0}.\end{aligned}$$

DEFINITION 1 System (1) is regularized by the feedback

$$u_k = Lx_{k+1} + w_k, \tag{3}$$

if there exist a gain $L \in \mathbb{R}^{m\times n}$ such that System (1) under the feedback (3) is equivalent to the system

$$E_r x_{k+1} = (A + \Delta A)x_k + (A_d + \Delta A_d)x_{k-d} + (B + \Delta B)w_k, \tag{4}$$

where E_r is a full rank matrix and $w_k \in \mathbb{R}^m$ is the new control input.

Remark that if the uncertainties ΔA, ΔB, ΔA_d are absent, then the dynamics of System (4) can be written as

$$x_{k+1} = E_r^{-1}Ax_k + E_r^{-1}A_d + x_{k-d} + E_r^{-1}Bw_k, \tag{5}$$

which means that all the properties of singular systems will disappear under the action of the regularizing feedback (3). If the uncertainties are present, rewriting System (4) in terms of separate known and unknown nominal matrices is not possible, but this will not prevent the search of a stabilizing controller as it will be shown in Section 3.

The regularization of System (1) by the feedback (3) turns on checking the invertibility of the matrix $E - (B + \Delta B)L$ for all possible uncertainties ΔB. This problem has not been considered until now and the method of seeking L constitutes the major difficulty since ΔB is not completely known. For this purpose, we will give sufficient LMI condition for the existence and computation of such gain.

The main result of this section is given in the following statement.

THEOREM 1 System (1) is regularized by the feedback

$$u_k = YP^{-1}x_{k+1} + w_k$$

or the matrix $E - (B + M_B F_B(k) N_B)YP^{-1}$ has a full rank if there exist a positive and definite matrix $P \in \mathbb{R}^{n\times n}$, a matrix $Y \in \mathbb{R}^{m\times n}$ and positive constant ϵ_b such that the following LMI holds:

$$\begin{bmatrix} PE' + EP - BY - Y'B' - \epsilon_b M_B M_B' & Y'N_B' & P \\ N_B Y & \epsilon_b I & \mathbf{0} \\ P & \mathbf{0} & P \end{bmatrix} > 0. \tag{6}$$

Proof. Let $B_\Delta = B + \Delta B$, then the matrix $E - B_\Delta Y P^{-1}$ has a full rank if

$$(E - B_\Delta Y P^{-1})' P^{-1} (E - B_\Delta Y P^{-1}) > 0. \tag{7}$$

The last inequality can be rewritten as follows:

$$(E' P^{-1} - P^{-1} Y' B_\Delta' P^{-1}) P (P^{-1} E - P^{-1} B_\Delta Y P^{-1}) > 0. \tag{8}$$

If we put $X = P^{-1}E - P^{-1}B_\Delta Y P^{-1}$ and $Z = I$, then using the result of Fact 1, we can write

$$X' P X + Z' P^{-1} Z \geqslant X' Z + Z' X, \tag{9}$$

or

$$\begin{aligned}&(E' P^{-1} - P^{-1} Y' B_\Delta' P^{-1}) P (P^{-1} E - P^{-1} B_\Delta Y P^{-1})\\ &\quad \geqslant E' P^{-1} - P^{-1} Y' B_\Delta' P^{-1} + P^{-1} E - P^{-1} B_\Delta Y P^{-1} - P^{-1}.\end{aligned}$$

If

$$E' P^{-1} - P^{-1} Y' B_\Delta' P^{-1} + P^{-1} E - P^{-1} B_\Delta Y P^{-1} - P^{-1} > 0, \tag{10}$$

then inequality (7) is satisfied, and hence the matrix $E - (B + M_B F_B(k) N_B) Y P^{-1}$ has a full rank. Pre- and post-multiplying inequality (10) by the matrix P, we obtain

$$P E' + E P - B_\Delta Y - Y' B_\Delta' - P > 0. \tag{11}$$

From the last inequality and using the result of Fact 1, we can write for some $\epsilon_b > 0$,

$$\begin{aligned}&P E' + E P - B_\Delta Y - Y' B_\Delta' - P\\ &\quad > P E' + E P - B Y - Y' B' - \epsilon_b M_B M_B' - \epsilon_b^{-1} Y' N_B' N_B Y - P.\end{aligned}$$

By the Schur complement lemma, the matrix

$$P E' + E P - B Y - Y' B' - \epsilon_b M_B M_B' - \epsilon_b^{-1} Y' N_B' N_B Y - P$$

is positive definite if the following LMI holds:

$$\begin{bmatrix} P E' + E P - B Y - Y' B' - \epsilon_b M_B M_B' - P & Y' N_B' \\ N_B Y & \epsilon_b I \end{bmatrix} > 0.$$

By the Schur complement, the last LMI is equivalent to (6). This ends the proof. □

3. Stabilizability

The aim of this section is to deal with the regularization and the quadratic stabilization of System (1) for all admissible uncertainties ΔA, ΔB and ΔA_d. The objective is to make the closed-loop system stable and non singular under the action of a feedback of the form $u_k = L x_{k+1} + K x_k$, where L and K are constant matrices to be determined. By the application of such feedback, the resulting system behaves as a stable system of the form

$$x_{k+1} = \mathscr{A}_c x_k + \mathscr{A}_d x_{k-d}, \tag{12}$$

where $\mathscr{A}_c$ and $\mathscr{A}_d$ are the closed-loop $n \times n$ dimensional matrices that are not necessarily known. By the action of the feedback part Lx_{k+1}, System (1) liberates from various phenomenons of singular systems and hence, a multi-objective controller design can be easily done by appropriate choice of the gain K. The design of the feedback gains is given by the following theorem.

THEOREM 2 Consider System (1). If there exist a positive and definite matrix $P \in \mathbb{R}^{n\times n}$, two matrices $Y_1 \in \mathbb{R}^{m\times n}$ and $Y_2 \in \mathbb{R}^{m\times n}$, and a set of positive constants ϵ_a, ϵ_b, $\tilde{\epsilon}_b$ and ϵ_d such that the following LMI holds:

$$\begin{bmatrix} \mathscr{M}_{1,1} & \mathbf{0} & PA' + Y_2'B' & \mathbf{0} & P & PN_A' & Y_2'N_B' & Y_1'N_B' \\ \mathbf{0} & -Q & QA_d' & QN_D' & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ AP + BY_2 & A_dQ & \mathscr{M}_{3,3} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & N_DQ & \mathbf{0} & -\epsilon_d I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ P & \mathbf{0} & \mathbf{0} & \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ N_AP & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\epsilon_a I & \mathbf{0} & \mathbf{0} \\ N_BY_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\epsilon_b I & \mathbf{0} \\ N_BY_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\tilde{\epsilon}_b I \end{bmatrix} < 0, \tag{13}$$

where $\mathscr{M}_{1,1} = -PE' - EP + BY_1 + Y_1'B' + \tilde{\epsilon}_b M_B M_B' + P$, $\mathscr{M}_{3,3} = -P + \epsilon_a M_A M_A' + \epsilon_d M_D M_D' + \epsilon_b M_B M_B'$. Then System (1) is asymptotically stable under the feedback

$$u_k = Y_1 P^{-1} x_{k+1} + Y_2 P^{-1} x_k, \tag{14}$$

and the matrix $E - (B + M_B F_B(k) N_B) Y P^{-1}$ has a full rank.

Proof. For notation simplicity, let $A_\Delta = A + \Delta A$, $B_\Delta = B + \Delta B$, $A_{d\Delta} = A_d + \Delta A_d$, $A_{c\Delta} = A_\Delta + B_\Delta Y_2 P^{-1}$, $E_{r\Delta} = E - B_\Delta Y_1 P^{-1}$. Under the action of controller (14), the dynamics of the closed-loop system is governed by

$$E_{r\Delta} x_{k+1} = A_{c\Delta} x_k + A_{d\Delta} x_{k-d}. \tag{15}$$

Taking the Lyapunov function candidate

$$V_k = x_k' E_{r\Delta}' P^{-1} E_{r\Delta} x_k + \sum_{i=k-d}^{k-1} x_i' Q^{-1} x_i, \tag{16}$$

we have

$$\begin{aligned} V_{k+1} - V_k &= x_{k+1}' E_{r\Delta}' P^{-1} E_{r\Delta} x_{k+1} - x_k' E_{r\Delta}' P^{-1} E_{r\Delta} x_k + x_k' Q^{-1} x_k - x_{k-d}' Q^{-1} x_{k-d} \\ &= x_k' (Q^{-1} - E_{r\Delta}' P^{-1} E_{r\Delta} + A_{c\Delta}' P^{-1} A_{c\Delta}) x_k + x_k' A_{c\Delta}' P^{-1} A_{d\Delta} x_{k-d} \\ &\quad + x_{k-d}' A_{d\Delta}' P^{-1} A_{c\Delta} x_k + x_{k-d}' A_{d\Delta}' P^{-1} A_{d\Delta} x_{k-d} - x_{k-d}' Q^{-1} x_{k-d}. \end{aligned}$$

The last difference $V_{k+1} - V_k$ can be rewritten as

$$\begin{bmatrix} x_k \\ x_{k-d} \end{bmatrix}' \begin{bmatrix} \mathscr{A}_{1,1} & \mathscr{A}_{1,2} \\ \mathscr{A}_{2,1} & \mathscr{A}_{2,2} \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-d} \end{bmatrix}, \tag{17}$$

where

$$\begin{aligned} \mathscr{A}_{1,1} &= Q^{-1} - E'_{r\Delta} P^{-1} E_{r\Delta} + A'_{c\Delta} P^{-1} A_{c\Delta}, \\ \mathscr{A}_{1,2} &= A'_{c\Delta} P^{-1} A_{d\Delta}, \\ \mathscr{A}_{2,1} &= \mathscr{A}'_{1,2}, \\ \mathscr{A}_{2,2} &= -Q^{-1} + A'_{d\Delta} P^{-1} A_{d\Delta}. \end{aligned}$$

System (1) is stable under the action of the feedback (14) if

$$\begin{bmatrix} \mathscr{A}_{1,1} & \mathscr{A}_{1,2} \\ \mathscr{A}_{2,1} & \mathscr{A}_{2,2} \end{bmatrix} < 0. \tag{18}$$

The last inequality is equivalent to the following LMI:

$$\begin{bmatrix} -PE'_{r\Delta} P^{-1} E_{r\Delta} P + PQ^{-1} P & \mathbf{0} & PA'_{c\Delta} \\ \mathbf{0} & -Q & QA'_{d\Delta} \\ A_{c\Delta} P & A_{d\Delta} Q & -P \end{bmatrix} < 0. \tag{19}$$

It is easy to verify that inequality (18) can be re-obtained by pre- and post-multiplying inequality (19) by the matrix

$$\begin{bmatrix} P^{-1} & \mathbf{0} & A'_{c\Delta} P^{-1} \\ \mathbf{0} & Q^{-1} & A'_{d\Delta} P^{-1} \end{bmatrix}. \tag{20}$$

We rewrite inequality (19) as

$$\begin{bmatrix} -PE'_{r\Delta} P^{-1} E_{r\Delta} P + PQ^{-1} P & \mathbf{0} & PA' + Y'_2 B' \\ \mathbf{0} & -Q & QA'_d \\ AP + BY_2 & A_d Q & -P \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & P\Delta A' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Delta AP & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q\Delta A'_d \\ \mathbf{0} & \Delta A_d Q & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & Y'_2 \Delta B' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Delta BY_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0.$$

Using the definitions of the uncertainties and the result of Fact 1, we have

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & P\Delta A' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Delta AP & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_A \end{bmatrix} F_A(k) [N_A P \quad \mathbf{0} \quad \mathbf{0}] + [N_A P \quad \mathbf{0} \quad \mathbf{0}]' F'_A(k) \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_A \end{bmatrix}'.$$

Using the result of Fact 1 with $F_A'(k)F_A(k) \leqslant I$, we can write that for any $\epsilon_a > 0$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & P\Delta A' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Delta AP & \mathbf{0} & \mathbf{0} \end{bmatrix} \leqslant \epsilon_a^{-1} \begin{bmatrix} PN_A' \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} [N_A P \quad \mathbf{0} \quad \mathbf{0}] + \epsilon_a \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_A \end{bmatrix} [\mathbf{0} \quad \mathbf{0} \quad M_A']$$

$$= \begin{bmatrix} \epsilon_a^{-1} PN_A' N_A P & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \epsilon_a M_A M_A' \end{bmatrix}.$$

Similarly,

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q\Delta A_d' \\ \mathbf{0} & \Delta A_d Q & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_D \end{bmatrix} F_D(k)[\mathbf{0} \quad N_d Q \quad \mathbf{0}] + [\mathbf{0} \quad N_d Q \quad \mathbf{0}]' F_D'(k) \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_D \end{bmatrix}'.$$

By the use of Fact 1 and taking into account $F_D'(k)F_D(k) \leqslant I$, then according to the last equality, we can deduce that for any $\epsilon_d > 0$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q\Delta A_d' \\ \mathbf{0} & \Delta A_d Q & \mathbf{0} \end{bmatrix} \leqslant \epsilon_d^{-1} \begin{bmatrix} \mathbf{0} \\ QN_D' \\ \mathbf{0} \end{bmatrix} [\mathbf{0} \quad N_D Q \quad \mathbf{0}] + \epsilon_d \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_D \end{bmatrix} [\mathbf{0} \quad \mathbf{0} \quad M_D']$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \epsilon_d^{-1} QN_D' N_D Q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \epsilon_d M_D M_D' \end{bmatrix}.$$

With the same analysis, we can write that for all $F_B'(k)F_B(k) \leqslant I$ and for all $\epsilon_b > 0$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & Y_2'\Delta B' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Delta BY_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_B \end{bmatrix} F_B(k)[N_B Y_2 \quad \mathbf{0} \quad \mathbf{0}] + [N_B Y_2 \quad \mathbf{0} \quad \mathbf{0}]' F_B'(k) \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_B \end{bmatrix}'$$

$$\leqslant \begin{bmatrix} \epsilon_b^{-1} Y_2' N_B' N_B Y_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \epsilon_b M_B M_B' \end{bmatrix}.$$

From the last development, we obtain a sufficient condition to fulfil inequality (19), i.e.

$$\begin{bmatrix} \mathscr{K}_{1,1} & \mathbf{0} & PA' + Y_2'B' \\ \mathbf{0} & \mathscr{K}_{2,2} & QA_d' \\ AP + BY_2 & A_d Q & \mathscr{K}_{3,3} \end{bmatrix} < 0, \tag{21}$$

where

$$\mathscr{K}_{1,1} = -PE'_{r\Delta}P^{-1}E_{r\Delta}P + PQ^{-1}P + \epsilon_a^{-1}PN'_AN_AP + \epsilon_b^{-1}Y'_2N'_BN_BY_2,$$

$$\mathscr{K}_{2,2} = -Q + \epsilon_d^{-1}QN'_DN_DQ,$$

$$\mathscr{K}_{3,3} = -P + \epsilon_aM_AM'_A + \epsilon_dM_DM'_D + \epsilon_bM_BM'_B.$$

Applying the Schur complement lemma, (21) is equivalent to

$$\begin{bmatrix} \mathscr{K}_{1,1} & \mathbf{0} & PA' + Y'_2B' & \mathbf{0} \\ \mathbf{0} & -Q & QA'_d & QN'_D \\ AP + BY_2 & A_dQ & \mathscr{K}_{3,3} & \mathbf{0} \\ \mathbf{0} & N_DQ & \mathbf{0} & -\epsilon_dI \end{bmatrix} < 0. \tag{22}$$

The last inequality is also equivalent by the Schur complement to the following inequality:

$$\begin{bmatrix} -PE'_{r\Delta}P^{-1}E_{r\Delta}P & \mathbf{0} & PA' + Y'_2B' & \mathbf{0} & P & PN'_A & Y'_2N'_B \\ \mathbf{0} & -Q & QA'_d & QN'_D & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ AP + BY_2 & A_dQ & \mathscr{M}_{3,3} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & N_DQ & \mathbf{0} & -\epsilon_dI & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ P & \mathbf{0} & \mathbf{0} & \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} \\ N_AP & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\epsilon_aI & \mathbf{0} \\ N_BY_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\epsilon_bI \end{bmatrix} < 0. \tag{23}$$

From the result of Theorem 1, we conclude that for $\tilde{\epsilon}_b > 0$,

$$-PE'_{r\Delta}P^{-1}E_{r\Delta}P < -PE' - EP + BY_1 + Y'_1B' + \tilde{\epsilon}_bM_BM'_B + \tilde{\epsilon}_b^{-1}Y'_1N'_BN_BY_1 + P. \tag{24}$$

From inequality (23) and using (24), the condition of stability of System (1) is equivalent to (13). This ends the proof. □

REMARK 1 According to (23), we remark that the negativity of the element $-PE'_{r\Delta}P^{-1}E_{r\Delta}P$ is a necessary condition. In the meantime, the condition $-PE'_{r\Delta}P^{-1}E_{r\Delta}P < 0$ is exactly the condition of the regularization of System (1), see inequality (8).

The LMI (13) gives a sufficient condition for regularization and stabilization of System (1) by means of combined memoryless and predictive feedback. Moreover, the LMI (13) is delay independent which makes the design of feedback gains, if they exist, valid for arbitrary amount of delay.

4. Illustrative example

To show the effectiveness of the developed results, consider the singular discrete-time delay system with the following nominal matrices:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & 5 & 1 \\ 3 & -2 & -1 \\ -1 & -1 & 1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.1 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.3 \\ -0.1 & -0.5 & 0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 4 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad M_A = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.2 & 0.1 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{bmatrix},$$

$$M_B = \begin{bmatrix} 2 & 1 \\ 1 & 0.9 \\ 2 & 1 \end{bmatrix}, \quad M_D = \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.02 & 0.01 & 0.3 \\ 0.1 & 0.1 & 0.3 \end{bmatrix},$$

$$N_A = I, \quad N_B = 0.1I, \quad N_D = 0.1I.$$

To solve LMI (13), we have used the LMI package of Matlab which gives us the following solution $\epsilon_a = 0.2974$, $\epsilon_b = 0.0293$, $\tilde{\epsilon}_b = 0.0289$, $\epsilon_d = 0.0402$ and

$$Q = \begin{bmatrix} 1.1757 & -0.2594 & -0.0421 \\ -0.2594 & 0.1345 & 0.0084 \\ -0.0421 & 0.0084 & 0.1233 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.5048 & 0.3193 & 0.4634 \\ 0.3193 & 0.2315 & 0.3114 \\ 0.4634 & 0.3114 & 0.5205 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -2.0931 & -0.0117 & 0.6577 \\ 0.6055 & -1.1859 & -3.0287 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -0.7771 & -0.6553 & -0.9839 \\ 1.1860 & 0.3696 & 0.2549 \end{bmatrix}.$$

5. Conclusion

In this paper, a new LMI condition for regularization of singular discrete systems with time delay and norm-bounded uncertainties is given. We showed that the action of regularizing feedback has permitted us to formulate the stabilizability condition of the singular system in terms of strict LMI. The benefits of adding a regularizing feedback is crucial to liberate from a lot of transient phenomenons of singular systems and hence, more optimality conditions on the design of the memoryless stabilizing feedback can be imposed. Observer-based control and regularization of this class of systems is under investigation and shall be the topic of our next contribution.

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