NEW LINEAR MATRIX INEQUALITIES CONDITIONS FOR OBSERVER-BASED STABILIZATION OF UNCERTAIN DISCRETE-TIME LINEAR SYSTEMS

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ABSTRACT

In this note, it is shown that the observer-based control of uncertain discrete-time linear systems is conditioned by the solvability of three linear matrix inequalities that must hold simultaneously. We show that the observer-based control problem, which is originally a non-convex issue, can be decomposed into two separate convex problems formulated as a set of numerically tractable linear matrix inequalities conditions. The new proposed linear matrix inequalities are neither iterative nor subject to any equality constraint. Illustrative example is given to indicate the novelty and effectiveness of the proposed design.

***Index Terms*—** Observer-based control; Discrete-time systems; System theory; Convex optimization; Linear Matrix Inequalities (LMIs).

1. INTRODUCTION

Usually, the design of feedback systems is achieved under the assumption that the system states are available for feedback. However, this unrealistic assumption is not always verified, and hence, the construction of the unmeasured states through the knowledge of the system inputs and outputs still an unavoidable task to solve any desired control problem. As a matter of fact, state estimation is not quite limited to stabilization exercises, but it is also a crucial task that permits to detect the system faults, evaluate the performances of industrial processes, or identify unknown parameters of inherently complex dynamical systems. The name of *observer* is referred to as a dynamical system that uses the information of the system inputs and outputs to reconstruct the unmeasured states of the system under consideration. For deterministic and stochastic linear systems, the theory of observers is well developed thanks to the pioneer works of Kalman [1] and Luenberger [2]. However, for uncertain linear systems, there is no generic procedure to solve the observation issue, which motivates the research in this area for the past decades, see for example [3]. When parts of the system dynamic is not completely known and the state vector is not entirely available for feedback, the available results are limited to some cases including matched uncertainties [4] norm-bounded uncertainties [5], [6] and uncertainties of dyadic types [7], [8].

The dynamic output feedback for discrete-time uncertain systems has been the subject of numerous papers, see e.g., [9], [10]. Reduced order observer-based compensators for continuous-time systems was discussed in [11]. Conceptually, the observer-based control of uncertain linear systems is recognized to be a non-convex issue since the computation of the observer and the controller gains is usually conditioned by the solution of some matrix inequalities which are not numerically tractable [12]. Available techniques that have been devoted to observer-based stabilization of uncertain linear systems can be classified into three categories: Lyapunov-based techniques as in [13], iterative linear matrix inequalities (ILMIs) procedures as proposed in [14], and convex-optimization-based algorithms with equality constraints as recently discussed in [15]. Roughly speaking, the Lyapunov-based design leads in general to complicated analysis and necessitates many computational steps to solve the entire problem. Even ILMIs algorithms give a straightforward method to solve the observer-based problem, the computational algorithms are at least two steps procedures that permit to find, in convex optimization setting, the observer and the controller gains. Therefore, ILMIs can not be classified as convex inequalities because they cannot be solved simultaneously. Linear matrix inequality algorithms subject to equality constraints as used in [15] permit to reverse the observer-based issue to a convex one but, in the meantime, they may increase the conservatism of the conditions under the presence of significant uncertainties. In this paper, new sufficient LMIs conditions, that guarantee the stability of discrete-time uncertain linear systems under the action of observer-based feedbacks, are proposed. By introducing new scalar variables, the original non-convex problem is decomposed into two separate convex issues: observer design and controller design. It will be

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shown that the determination of the observer and the controller gains is conditioned by the solvability of three linear matrix inequalities that must hold simultaneously. In comparison with existing results, the proposed LMIs are neither subject to any equality constraint nor iterative. Furthermore, the proposed design is novel in the sense that the observer design issue is decoupled from the controller design problem by introduction of new free parameters that link the two separate problems. The novelty of the proposed numerical procedure is tested through an example. Throughout this paper, the notation $A > 0$ (respectively $A < 0$) means that the matrix A is positive definite (respectively negative definite). We denote by A' the matrix transpose of A. We note by I and $\mathbf{0}$ the identity matrix, and the null matrix of appropriate dimensions, respectively. $\mathbb{R}$ stands for the set of real numbers, and "$\star$" is used to notify a matrix element that is induced by transposition.

2. OBSERVER-BASED CONTROL OF DISCRETE-TIME UNCERTAIN SYSTEMS

2.1. System description and preliminaries

Consider the uncertain linear system

$$\begin{aligned} x_{k+1} &= (A + \Delta A(k))\, x_k + (B + \Delta B(k))\, u_k, \\ y_k &= (C + \Delta C(k))\, x_k + (D + \Delta D(k))\, u_k, \end{aligned} \quad (1)$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input, and $y_k \in \mathbb{R}^p$ is the system output. The nominal matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $D \in \mathbb{R}^{p\times m}$ are constant known matrices and $\Delta A(k) \in \mathbb{R}^{n\times n}$, $\Delta B(k) \in \mathbb{R}^{n\times m}$, $\Delta C(k) \in \mathbb{R}^{p\times n}$, and $\Delta D(k) \in \mathbb{R}^{p\times m}$ are partially known uncertainties defined as follows

$$\begin{aligned} \Delta A(k) &= M_A\, F_A(x_k, k)\, N_A; \quad F_A'(x_k, k) F_A(x_k, k) \le I, \\ \Delta B(k) &= M_B\, F_B(x_k, k)\, N_B; \quad F_B'(x_k, k) F_B(x_k, k) \le I, \\ \Delta C(k) &= M_C\, F_C(x_k, k)\, N_C; \quad F_C'(x_k, k) F_C(x_k, k) \le I, \\ \Delta D(k) &= M_D\, F_D(x_k, k)\, N_D; \quad F_D'(x_k, k) F_D(x_k, k) \le I, \end{aligned} \quad (2)$$

where $M_A \in \mathbb{R}^{n\times n}$, $N_A \in \mathbb{R}^{n\times n}$, $M_B \in \mathbb{R}^{n\times m}$, $N_B \in \mathbb{R}^{m\times m}$, $M_C \in \mathbb{R}^{p\times n}$, $N_C \in \mathbb{R}^{n\times n}$, $M_D \in \mathbb{R}^{p\times m}$ and $N_D \in \mathbb{R}^{m\times m}$ are known matrices and $F_A(x_k, k)$, $F_B(x_k, k)$, $F_C(x_k, k)$ and $F_D(x_k, k)$ are some unknown matrices of appropriate dimensions. We assume that the pair (A, B) is controllable and the pair (A, C) is observable. The result of the Schur complement lemma is used to prove the main result of this paper. Therefore, we would rather recall it [16].

Lemma 1 *Given constant matrices M, N, Q of appropriate dimensions where M and Q are symmetric, then $Q > 0$ and $M + N'Q^{-1}N < 0$ if and only if* $\begin{bmatrix} M & N' \\ N & -Q \end{bmatrix} < 0$, *or equivalently* $\begin{bmatrix} -Q & N \\ N' & M \end{bmatrix} < 0$.

Fact 1 *For given matrices X and Y with appropriate dimensions, we have*

$$X'Y + Y'X \le \varepsilon X'X + \varepsilon^{-1} Y'Y, \quad \varepsilon > 0. \quad (3)$$

The following lemma is useful for the next derivations.

Lemma 2 *Let $P > 0$ be a symmetric and positive definite matrix, and let α and β be two positive reals. Then,*

$$P > \frac{\alpha}{\beta^2} I \quad (4)$$

holds, if the following linear matrix inequality holds

$$\begin{bmatrix} P & I \\ I & (2\beta - \alpha) I \end{bmatrix} > 0. \quad (5)$$

Proof. By the Schur complement, the matrix inequality (4) is equivalent to the following matrix inequality

$$\begin{bmatrix} P & I \\ I & \frac{\beta^2}{\alpha} I \end{bmatrix} > 0. \quad (6)$$

Since for any $\alpha > 0$ and $\beta > 0$, we have $\frac{1}{\alpha}\Big(\alpha I - \beta I\Big)\Big(\alpha I - \beta I\Big) \ge 0$. Then, by expanding the last inequality, we get

$$\frac{\beta^2}{\alpha} I \ge (2\beta - \alpha) I. \quad (7)$$

By the use of (6) and (7), we get (5).

2.2. Special case of uncertainties

Consider the following discrete-time system

$$\begin{aligned} x_{k+1} &= (A + \Delta A(k))\, x_k + B\, u_k, \\ y_k &= (C + \Delta C(k))\, x_k, \end{aligned} \quad (8)$$

where the nominal matrices are defined as in subsection 2.1. System (8) is a special case of system (1) where $\Delta B = 0$, $D = \Delta D(k) = 0$. For system (8), we develop an observer of the following form

$$\hat{x}_{k+1} = A\hat{x}_k + B\, u_k + L\Big(C\hat{x}_k - y_k\Big), \quad (9)$$

where L stands for the observer gain. The objective of this subsection is to find the observer gain $L \in \mathbb{R}^{n\times p}$ and the controller gain $K \in \mathbb{R}^{m\times n}$ such that system (8) is globally asymptotically stable under the action of the linear feedback $u_k = K\hat{x}_k$.

In this section, we shall propose new sufficient LMIs conditions that are not subject to any equality constraint and enjoy the property to be numerically tractable by any convex optimization software. We summarize the design of the observer-based controller in the following statement.

Theorem 1 *Consider system (8) and observer (9). If there exist two symmetric and positive definite matrices $P_1 \in \mathbb{R}^{n\times n}$, $P_2 \in \mathbb{R}^{n\times n}$, two real matrices $Y_1 \in \mathbb{R}^{m\times n}$, $Y_2 \in \mathbb{R}^{n\times p}$ and five positive constants α, β, ε_1, ε_2 and ε_3 such that the following hold*

$$\begin{bmatrix} P_1 & I \\ \star & (2\beta-\alpha)I \end{bmatrix} > 0, \tag{10}$$

$$\begin{bmatrix} -P_1+\varepsilon_3 M_A M_A' & AP_1+BY_1 & BY_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & -P_1 & \mathbf{0} & P_1N_A' & P_1N_C' & P_1N_A' \\ \star & \star & -\alpha I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -\varepsilon_1 I & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & \star & -\varepsilon_2 I & \mathbf{0} \\ \star & \star & \star & \star & \star & -\varepsilon_3 I \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} -P_2 & A'P_2+C'Y_2' & \beta I & \mathbf{0} & \mathbf{0} \\ \star & -P_2 & \mathbf{0} & P_2M_A & Y_2M_C \\ \star & \star & -P_1 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -(2-\varepsilon_1)I & \mathbf{0} \\ \star & \star & \star & \star & -(2-\varepsilon_2)I \end{bmatrix} < 0, \tag{12}$$

then there exist two gains $L = P_2^{-1}Y_2$ and $K = Y_1P_1^{-1}$ such that systems (8) and (9) are globally asymptotically stable under the feedback $u_k = K\hat{x}_k$.

Proof. Let $A_\Delta = A + \Delta A(k)$. Denoting $e_k = \hat{x}_k - x_k$ then, we can form the composite system

$$\begin{aligned} x_{k+1} &= (A_\Delta + B\,K)x_k + B\,Ke_k, \\ e_{k+1} &= (A+L\,C)e_k - (\Delta A(k) + L\,\Delta C)x_k. \end{aligned} \tag{13}$$

Let us associate to the dynamics (13) the following Lyapunov function $V_k = x_k'P_1^{-1}x_k + e_k'P_2e_k$. Then, $\Delta V_k = V_{k+1} - V_k$ is given by

$$\begin{aligned} \Delta V_k &= x_k'\Big[(A_\Delta + B\,K)'P_1^{-1}(A_\Delta + B\,K) - P_1^{-1}\Big]x_k \\ &\quad + x_k'(A_\Delta + B\,K)'P_1^{-1}B\,Ke_k \\ &\quad + e_k'K'\,B'P_1^{-1}(A_\Delta + B\,K)x_k \\ &\quad + e_k'K'\,B'P_1^{-1}B\,Ke_k \\ &\quad + \Big[e_k'(A+L\,C)' - x_k'(\Delta A(k) + L\,\Delta C)'\Big]P_2 \\ &\quad \times \Big[(A+L\,C)e_k - (\Delta A + L\,\Delta C(k))x_k\Big] - e_k'P_2e_k. \end{aligned} \tag{14}$$

The difference $V_{k+1} - V_k$ can be rewritten in matrix form as follows

$$V_{k+1} - V_k = \begin{bmatrix} x_k \\ e_k \end{bmatrix}' \begin{bmatrix} W_{11} & W_{12} \\ W_{12}' & W_{22} \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix}, \tag{15}$$

where

$$\begin{aligned} W_{11} &= -P_1^{-1} + (A_\Delta + B\,K)'P_1^{-1}(A_\Delta(k) + B\,K) \\ &\quad + (\Delta A(k) + L\,\Delta C(k))'P_2(\Delta A(k) + L\,\Delta C(k)), \\ W_{12} &= (A_\Delta + B\,K)'P_1^{-1}B\,K \\ &\quad - (\Delta A(k) + L\,\Delta C(k))'P_2(A + L\,C), \\ W_{22} &= -P_2 + K'\,B'P_1^{-1}B\,K + (A+L\,C)'P_2(A+L\,C). \end{aligned} \tag{16}$$

The difference $V_{k+1} - V_k < 0$, if the following holds

$$\begin{bmatrix} W_{11} & W_{12} \\ W_{12}' & W_{22} \end{bmatrix} < 0. \tag{17}$$

The last matrix inequality is equivalent by the Schur complement to

$$\begin{bmatrix} \Pi_{11} & -(\Delta A(k) + L\,\Delta C(k))'P_2(A+L\,C) & (A_\Delta + B\,K)' \\ \star & -P_2 + (A+L\,C)'P_2(A+L\,C) & (B\,K)' \\ \star & \star & -P_1 \end{bmatrix} < 0, \tag{18}$$

where $\Pi_{11} = -P_1^{-1} + (\Delta A(k) + L\,\Delta C(k))'P_2(\Delta A(k) + L\,\Delta C(k))$. (18) is equivalent by the Schur complement to the following matrix inequality

$$\begin{bmatrix} -P_1^{-1} & \mathbf{0} & -(\Delta A(k) + L\,\Delta C(k))'P_2 & (A_\Delta(k) + B\,K)' \\ \star & -P_2 & (A+L\,C)'P_2 & (B\,K)' \\ \star & \star & -P_2 & \mathbf{0} \\ \star & \star & \star & -P_1 \end{bmatrix} < 0.$$

Pre- and post multiplying the last inequality by the matrix $\mathrm{diag}\Big(P_1, I, I, I\Big)$, we obtain the following inequality

$$\begin{bmatrix} -P_1 & \mathbf{0} & -P_1(\Delta A(k) + L\,\Delta C(k))'P_2 & P_1(A_\Delta + B\,K)' \\ \star & -P_2 & (A+L\,C)'P_2 & (B\,K)' \\ \star & \star & -P_2 & \mathbf{0} \\ \star & \star & \star & -P_1 \end{bmatrix} < 0. \tag{19}$$

By the Schur complement, inequality (19) is equivalent to

$$\begin{bmatrix} -P_2 & (A+LC)'P_2 & (BK)' & \mathbf{0} \\ \star & -P_2 & \mathbf{0} & -P_2(\Delta A(k)+L\Delta C(k))P_1 \\ \star & \star & -P_1 & (A_\Delta + BK)P_1 \\ \star & \star & \star & -P_1 \end{bmatrix} < 0, \qquad (20)$$

which is also equivalent by the Schur complement to the following matrix inequality

$$\begin{bmatrix} -P_1 & (A_\Delta + BK)P_1 & BK & \mathbf{0} \\ \star & -P_1 & \mathbf{0} & \mathbf{0} \\ \star & \star & -P_2 & (A+LC)'P_2 \\ \star & \star & \star & -P_2 \end{bmatrix}
- \begin{bmatrix} \mathbf{0} \\ P_1 N_A' \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} F_A'(x_k,k) \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & M_A' P_2 \end{bmatrix}
- \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ P_2 M_A \end{bmatrix} F_A(x_k,k) \begin{bmatrix} \mathbf{0} & N_A P_1 & \mathbf{0} & \mathbf{0} \end{bmatrix}
- \begin{bmatrix} \mathbf{0} \\ P_1 N_C' \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} F_C'(x_k,k) \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & M_C' Y_2' \end{bmatrix}
- \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ Y_2 M_C \end{bmatrix} F_C(x_k,k) \begin{bmatrix} \mathbf{0} & N_C P_1 & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0. \qquad (21)$$

Using Fact 1, we can write that (21) is satisfied if there exist $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that the following holds

$$\begin{bmatrix} -P_1 & (A_\Delta + BK)P_1 & BK & \mathbf{0} \\ \star & S_{22} & \mathbf{0} & \mathbf{0} \\ \star & \star & -P_2 & (A+LC)'P_2 \\ \star & \star & \star & S_{44} \end{bmatrix} < 0, \qquad (22)$$

where $S_{22} = -P_1 + \varepsilon_1^{-1} P_1 N_A' N_A P_1 + \varepsilon_2^{-1} P_1 N_C' N_C P_1$, $S_{44} = -P_2 + \varepsilon_1 P_2 M_A M_A' P_2 + \varepsilon_2 Y_2 M_C M_C' Y_2'$. Then, for any $\alpha > 0$, inequality (22) can be rewritten as follows

$$\begin{bmatrix} I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & P_1^{-1} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \times
\begin{bmatrix} -P_1 & (A_\Delta + BK)P_1 & BY_1 & \mathbf{0} & \mathbf{0} \\ \star & S_{22} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & -\alpha I & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -P_2 + \alpha P_1^{-1} P_1^{-1} & (A+LC)'P_2 \\ \star & \star & \star & \star & S_{44} \end{bmatrix} \times
\begin{bmatrix} I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & P_1^{-1} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix}' < 0. \qquad (23)$$

This immediately implies that sufficient conditions to fulfill condition (22) are

$$\begin{bmatrix} -P_1 & (A_\Delta + BK)P_1 & BY_1 \\ \star & S_{22} & \mathbf{0} \\ \star & \star & -\alpha I \end{bmatrix} < 0, \qquad (24)$$

$$\begin{bmatrix} -P_2 + \alpha P_1^{-1} P_1^{-1} & (A+LC)'P_2 \\ \star & S_{44} \end{bmatrix} < 0. \qquad (25)$$

From (24), we have

$$\begin{bmatrix} -P_1 & AP_1 + BY_1 & BY_1 \\ \star & S_{22} & \mathbf{0} \\ \star & \star & -\alpha I \end{bmatrix}
+ \begin{bmatrix} M_A \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} F_A(x_k,k) \begin{bmatrix} \mathbf{0} & N_A P_1 & \mathbf{0} \end{bmatrix}
+ \begin{bmatrix} \mathbf{0} & N_A P_1 & \mathbf{0} \end{bmatrix}' F_A'(x_k,k) \begin{bmatrix} M_A \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}' < 0. \qquad (26)$$

Then, by the use of Fact 1, we can write that a sufficient condition to fulfill the last matrix inequality is to find some $\varepsilon_3 > 0$ such that

$$\begin{bmatrix} -P_1 + \varepsilon_3 M_A M_A' & AP_1 + BY_1 & BY_1 \\ \star & S_{22} + \varepsilon_3^{-1} P_1 N_A' N_A P_1 & \mathbf{0} \\ \star & \star & -\alpha I \end{bmatrix} < 0,$$

which is equivalent by the Schur complement to the LMI (11). In order to make inequality (25) linear with respect to their variable, let $\beta > 0$ be some independent constant such that

$$P_1 > \frac{\alpha}{\beta^2} I. \qquad (27)$$

Then, by the use of result of Lemma 2, we can then deduce that (27) holds if the following LMI holds

$$\begin{bmatrix} P_1 & I \\ I & (2\beta-\alpha)I \end{bmatrix} > 0. \tag{28}$$

From (27) and (25), we derive a new sufficient condition to fulfill (25), that is

$$\begin{bmatrix} -P_2+\beta^2 P_1^{-1} & A'P_2+C'Y_2' \\ \star & \mathscr{L}_{22} \end{bmatrix} < 0, \tag{29}$$

where $\mathscr{L}_{22} = -P_2 + \varepsilon_1 P_2 M_A M_A' P_2 + \varepsilon_2 Y_2 M_C M_C' Y_2'$. By the Schur complement, the last matrix inequality is equivalent to

$$\begin{bmatrix} -P_2 & A'P_2+C'Y_2' & \beta I & \mathbf{0} & \mathbf{0} \\ \star & -P_2 & \mathbf{0} & P_2 M_A & Y_2 M_C \\ \star & \star & -P_1 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -\varepsilon_1^{-1} I & \mathbf{0} \\ \star & \star & \star & \star & -\varepsilon_2^{-1} I \end{bmatrix} < 0. \tag{30}$$

Since for any $\varepsilon > 0$, we have $\varepsilon^{-1}(I-\varepsilon I)(I-\varepsilon I) \geq 0$ or equivalently, $-\varepsilon^{-1} I \leq -(2-\varepsilon)I$. Then, the last matrix inequality condition holds if (12) holds. This ends the proof.

Remark 1 *During the derivation, the coefficient α is introduced so as to dissociate the original non-convex problem into two separate problems: controller design and observer design. Hereafter, the coefficient β is introduced in order to make the LMI (25) linear with respect to their variables. Inequality (10) describes the link between (11) and (12).*

Remark 2 *The passage from the matrix inequality (22) to inequalities (24) and (25) is certainly paid by certain conservatism. However, the appearance of the new parameters α and β reduces the conservatism of the sufficient conditions.*

2.3. Case of uncertainties in all state matrices

Consider now system (1) that represents the general case where uncertainties are present in all the state matrices. In this subsection, we show by state augmentation that the problem of observer-based control of system (1) turns out to be a stabilization problem of an augmented system of form (8). To this end, let us consider the new state variables $\xi_k = \begin{bmatrix} x_k \\ u_k \end{bmatrix}$, with $v_k = u_{k+1}$ is the new control input. Then, the dynamics of the ξ_k-system becomes

$$\begin{aligned} \xi_{k+1} &= (\boldsymbol{A} + \Delta\boldsymbol{A}(k))\,\xi_k + \boldsymbol{B}\,v_k, \\ y_k &= (\boldsymbol{C} + \Delta\boldsymbol{C}(k))\,\xi_k, \end{aligned} \tag{31}$$

where

$$\begin{aligned} &\boldsymbol{A} = \begin{bmatrix} A & B \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{I} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} C & D \end{bmatrix}, \\ &\Delta\boldsymbol{A}(k) = \begin{bmatrix} \Delta A(k) & \Delta B(k) \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ &\Delta\boldsymbol{C}(k) = \begin{bmatrix} \Delta C(k) & \Delta D(k) \end{bmatrix}. \end{aligned} \tag{32}$$

The resulting uncertainties take the initial forms $\Delta\boldsymbol{A}(k) = \boldsymbol{M}_A \boldsymbol{F}_A(x_k,k)\boldsymbol{N}_A$ and $\Delta\boldsymbol{C}(k) = \boldsymbol{M}_C \boldsymbol{F}_C(x_k,k)\boldsymbol{N}_C$ where

$$\begin{aligned} &\boldsymbol{M}_A = \begin{bmatrix} M_A & M_B \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \boldsymbol{N}_A = \begin{bmatrix} N_A & \mathbf{0} \\ \mathbf{0} & N_B \end{bmatrix}, \\ &\boldsymbol{M}_C = \begin{bmatrix} M_C & M_D \end{bmatrix}, \ \boldsymbol{N}_C = \begin{bmatrix} N_C & \mathbf{0} \\ \mathbf{0} & N_D \end{bmatrix}, \\ &\boldsymbol{F}_A(x_k,k) = \begin{bmatrix} F_A(x_k,k) & \mathbf{0} \\ \mathbf{0} & F_B(x_k,k) \end{bmatrix}, \\ &\boldsymbol{F}_C(x_k,k) = \begin{bmatrix} F_C(x_k,k) & \mathbf{0} \\ \mathbf{0} & F_D(x_k,k) \end{bmatrix}. \end{aligned} \tag{33}$$

3. ILLUSTRATIVE EXAMPLE

To illustrate the powerfulness of the proposed LMIs, let us consider the discrete-time system with the following state matrices

$$\begin{aligned} &A = \begin{bmatrix} 1 & 0.1 & 0.4 \\ 1 & 1 & 0.5 \\ -0.3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.1 & 0.3 \\ -0.4 & 0.5 \\ 0.6 & 0.4 \end{bmatrix}, \\ &C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, M_A = \begin{bmatrix} 0 & 0 & 0 \\ 0.1 & 0.3 & 1 \\ 0 & 0.2 & 0 \end{bmatrix}, \\ &N_A = \begin{bmatrix} 0 & 0 & 0 \\ 0.2 & 0 & 0.4 \\ 0 & 0.1 & 0 \end{bmatrix}, M_C = \begin{bmatrix} 0 & 0 & 0.3 \\ 0 & 0 & 0.8 \end{bmatrix}, \\ &N_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}. \end{aligned} \tag{34}$$

By the use of the Matlab LMI package, a solution of LMIs of Theorem 1 is

$$\begin{aligned}
P_1 &= \begin{bmatrix} 1.8720 & -0.7033 & -0.5523 \\ -0.7033 & 4.9703 & -0.5303 \\ -0.5523 & -0.5303 & 1.5312 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 6.1099 & -0.2091 & 0.1894 \\ -0.2091 & 0.6135 & 0.1735 \\ 0.1894 & 0.1735 & 2.0335 \end{bmatrix}, \\
Y1 &= \begin{bmatrix} 1.1652 & 1.1241 & -1.2826 \\ -1.2499 & -1.1599 & -0.7464 \end{bmatrix}, \\
Y_2 &= \begin{bmatrix} -2.3531 & -0.5679 \\ 0.2228 & -0.5966 \\ -1.8613 & -0.0019 \end{bmatrix}, \\
&\varepsilon_1 = 0.7964,\ \varepsilon_2 = 0.7422,\ \varepsilon_3 = 0.7046,\ \alpha = 1.7777, \\
&\beta = 1.4426.
\end{aligned} \tag{35}$$

4. CONCLUSION

New sufficient linear matrix inequality conditions are proposed to solve the dynamic output feedback for discrete-time systems with norm-bounded uncertainties. The proposed design is novel, in the sense that, the numerically tractable conditions are neither iterative nor subject to equality constraints. Furthermore, the proposed design is straightforward and covers general systems with uncertainties in all the state matrices.

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