STABILITY AND ROBUST STABILIZATION OF DISCRETE-TIME SWITCHED SYSTEMS WITH TIME-DELAYS: LMI APPROACH

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ABSTRACT

We present sufficient linear matrix inequality conditions for asymptotic stability and stabilizability of switched discrete-time linear systems subject to time delays and norm bounded uncertainties. Namely, if these LMIs are solvable then, the switched system is exponentially stable for arbitrary switching. In fact, we show that any family of switched time-delay systems satisfying these conditions possesses a quadratic common Lyapunov function. We also discuss the implication of this result on the stabilizability of this class of systems by switching controllers that use common Lyapunov functions. We show that even the analysis is carried out through a common Lyapunov function, the applied controller is mode dependent.

***Index Terms*—** Switched systems; Discrete-time systems; Convex optimization; Time-delay systems; System theory.

1. INTRODUCTION

Dynamical properties of numerous physical systems are traditionally described by multiple dynamical models. This description comes from the fact that systems parameters may be subject to abrupt changes or simply control actions are often organized in a logical manner. Examples of practical multiple-model systems appear in numerous engineering fields as robotics, power systems, and aerospace technology. Therefore, the study of these systems has received a considerable attention during the last decade.

By a *switched model*, we mean a hybrid dynamical model that is composed of a set of subsystems and a rule orchestrating the switching between the subsystems. Recently, an increasing interest has been devoted to the study of such systems in both continuous-time and discrete-time cases, see for example [1], [2], [3], [4], [5], [6] and the references therein. The first main question that motivates the research in this area is the stability and the stabilizability problems. Since the property of stability forms a fundamental base for most breakdown methods in control theory, stability conditions for switched systems have been studied by many authors. As Lyapunov theory provides a convenient starting point for the study of such systems, there were essentially two ways to stability analysis. The first one involves seeking of a common a Lyapunov function and the other involves construction of multiple Lyapunov functions. The former task is usually more challenging, though once a common Lyapunov function is found, the breakdown of stability and stabilizability becomes quite simple. The second method, even it involves arduous analysis, still more amenable to numerous applications.

Robust stability and stabilizability of single-mode discrete-time systems with delays were addressed in many works and solutions to these problems were generally attached to some nonlinear optimization techniques, see for example [7]. The reader is also referred to the references [8], [9], [10], [11], [12], [13], [14], for further details on single-mode time-delay stabilization. Actually, the available results on robust stabilizability of delay systems can be classified into two groups: delay-dependent stabilization approach like in [11] and delay-independent stabilization techniques like in [13]. The available LMI-based results on robust stabilization of discrete-time delay systems are delay-dependent techniques, see for example [11]. The goal of this paper is to find, in terms of LMIs, delay-independent conditions for robust stability and stabilizability of uncertain discrete-time switched systems that contain switches in all the nominal matrices including the delay matrix. In fact, the switch in the delay matrix makes the problem of finding sufficient LMIs conditions so difficult as it was mentioned in many contributions that deal with single-mode systems. In this paper, we give sufficient LMIs conditions for the existence of a common Lyapunov functions for robust stability of uncertain time-delay switched systems. The developed LMIs conditions will help us later to deliver the expressions of switching controllers that guarantee the stabilizability of the considered system with a common Lyapunov function. We show the implication of the LMI stability conditions in the computation of the switching feedbacks gains that

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guarantee the asymptotic stability of the uncertain time-delay switched system. In other words, the stabilizability LMIs conditions are obtained by replacing the nominal state matrices, appearing in the stability LMIs conditions, by the closed-loop state matrices that involve new gains to be determined.

Throughout this paper we note by $\mathbb{R}$ and $\mathbb{Z}$, the set of real and integer numbers, respectively. $A > 0$ (resp. $A < 0$) means that the matrix A is positive definite (resp. negative definite). A' is the transpose of the matrix A. I is the identity matrix with appropriate dimensions and **0** stands for the null matrix with appropriate dimensions.

2. STABILITY

2.1. Stability without uncertainties

Consider the time-delay switched linear system

$$\begin{array}{l} x_{k+1} = A(\sigma)x_k + A_d(\sigma)x_{k-d} + B(\sigma)u_k, \\ y_k = C(\sigma)x_k, \end{array} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the system input and $y_k \in \mathbb{R}^p$ stands for the switching system output. σ is a piecewise constant switching signal. We assume that $x_{k-d} = \phi(k)$ for $k - d \leq 0$, $k \in \mathbb{Z}$. d is a positive parameter representing the amount of delay. The switched system admits "s" possible configurations $(A(j))_{1\leq j\leq s}$, $(B(\sigma))_{1\leq j\leq s}$, and $C(\sigma)_{1\leq j\leq s}$ and the switch between the nominal matrices is assumed to be arbitrary. We note by $\mathscr{S} = \{1, 2, \cdots, s\}$ the set of the "s" possible modes of system (1). The switching instants are supposed to be known by an appropriate mechanism. The stability conditions of the switched system (1) is given in the following statement.

Theorem 1 *Consider system (1) with $u_k \equiv 0$. If there exist two symmetric and positive definite matrices $X \in \mathbb{R}^{n\times n}$, and $Z \in \mathbb{R}^{n\times n}$ such that for all $j \in \mathscr{S}$, we have*

$$\begin{array}{c} \begin{bmatrix} -X & X & XA'(j) \\ X & -Z & \mathbf{0} \\ A(j)X & \mathbf{0} & -X + A_d(j)ZA_d'(j) \end{bmatrix} < 0, \\ \begin{bmatrix} -Z & ZA_d'(j) \\ A_d(j)Z & -X \end{bmatrix} < 0, \end{array} \tag{2}$$

then, system (1) is asymptotically stable under arbitrary switching, and $V_k = x_k' X^{-1} x_k + \sum_{i=k-d}^{k-1} x_i' Z^{-1} x_i$ is a common Lyapunov function for the switched system (1).

Proof. Let $\Delta V_k = V_{k+1} - V_k$ then, we have

$$\begin{aligned} V_{k+1} - V_k &= x_{k+1}' X^{-1} x_{k+1} - x_k' X^{-1} x_k \\ &\quad + \sum_{i=k-d+1}^{k} x_i' Z^{-1} x_i - \sum_{i=k-d}^{k-1} x_i' Z^{-1} x_i \\ &= x_{k-d}' A_d'(\sigma) X^{-1} A(\sigma) x_k \\ &+ x_{k-d}' A_d'(\sigma) X^{-1} A_d(\sigma) x_{k-d} \\ &+ x_k' A'(\sigma) X^{-1} A(\sigma) x_k \\ &+ x_k' A'(\sigma) X^{-1} A_d(\sigma) x_{k-d} \\ &- x_k' X^{-1} x_k - x_{k-d}' Z^{-1} x_{k-d} \\ &+ x_k' Z^{-1} x_k. \end{aligned} \tag{3}$$

Under the assumption that the matrices

$$\Big(Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma)\Big) > 0, \tag{4}$$

for all $\sigma \in \mathscr{S}$ then, we can write

$$\begin{aligned} \Delta V_k = x_k' \Bigg(& - X^{-1} + A'(\sigma) X^{-1} A(\sigma) \\ & + A'(\sigma) X^{-1} A_d(\sigma) \Big(Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma)\Big)^{-1} \\ & A_d'(\sigma) X^{-1} A(\sigma) + Z^{-1} \Bigg) x_k \\ & - \Bigg(\Big(Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma)\Big)^{-1} A_d'(\sigma) X^{-1} A(\sigma) x_k \\ & - x_{k-d} \Bigg)' \Big(Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma)\Big) \times \\ & \Bigg(\Big(Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma)\Big)^{-1} A_d'(\sigma) X^{-1} A(\sigma) x_k \\ & - x_{k-d} \Bigg). \end{aligned}$$

From the last equations, we conclude that $\Delta V_k < 0$ if

$$\begin{aligned} & - X^{-1} + A'(\sigma) X^{-1} A(\sigma) + A'(\sigma) X^{-1} A_d(\sigma) \\ & \Big(Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma)\Big)^{-1} A_d'(\sigma) X^{-1} A(\sigma) + Z^{-1} < 0, \end{aligned} \tag{5}$$

and

$$Z^{-1} - A_d'(\sigma) X^{-1} A_d(\sigma) > 0 \tag{6}$$

for all $\sigma \in \mathscr{S}$. Pre- and post multiplying inequality (6) by Z, we obtain

$$-Z + Z A_d'(\sigma) X^{-1} A_d(\sigma) Z < 0. \tag{7}$$

By the use of the inversion lemma, we have

$$\Big(-X + A_d(\sigma)ZA_d^{'}(\sigma)\Big)^{-1} = -X^{-1} \\ - X^{-1}A_d(\sigma)\Big(Z^{-1} - A_d^{'}(\sigma)X^{-1}A_d(\sigma)\Big)^{-1}A_d^{'}(\sigma)X^{-1} \tag{8}$$

then, inequality (5) is equivalent to

$$-X^{-1} + Z^{-1} - A'(\sigma)\Big(-X + A_d(\sigma)ZA_d^{'}(\sigma)\Big)^{-1}A(\sigma) \\ < 0. \tag{9}$$

Pre- and post multiplying the last inequality by X, we obtain

$$-X + XZ^{-1}X \\ -XA'(\sigma)\Big(-X + A_d(\sigma)ZA_d^{'}(\sigma)\Big)^{-1}A(\sigma)X < 0. \tag{10}$$

Using the Schur complement lemma, we can easily show that inequalities (10) and (7) are equivalent to conditions (2). This ends the proof.

2.2. Stability with uncertainties

In this subsection, we analyze the stability of switched systems with uncertainties in both the state and the state delay matrices. The conditions of stability are formulated in LMIs framework which can be easily solved by any LMI software. The results given herein will serve as a starting point to give sufficient LMIs conditions for the stabilizability of this class of systems by dynamic memoryless switching controllers. Consider the uncertain switched system

$$\begin{aligned} x_{k+1} &= (A(\sigma) + \Delta A(k,\sigma))\,x_k \\ &+ (A_d(\sigma) + \Delta A_d(k,\sigma)))\,x_{k-d} \\ &+ (B(\sigma) + \Delta B(k,\sigma))\,u_k, \end{aligned} \tag{11}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the system input and $x_{k-d} = \varphi(k)$ for $k - d \leq 0$. The uncertain terms $\Delta A(k,\sigma) \in \mathbb{R}^{n\times n}$, $\Delta A_d(k,\sigma) \in \mathbb{R}^{n\times n}$, and $\Delta B(k,\sigma) \in \mathbb{R}^{n\times m}$ are defined as

$$\begin{cases} \Delta A(k,\sigma) = M_A(\sigma)F_A(k)N_A(\sigma), \\ \Delta A_d(k,\sigma) = M_d(\sigma)F_d(k)N_d(\sigma), \\ \Delta B(k,\sigma) = M_B(\sigma)F_B(k)N_B(\sigma), \end{cases} \tag{12}$$

where $M_A(\sigma) \in \mathbb{R}^{n\times n}$, $M_B(\sigma) \in \mathbb{R}^{n\times m}$, $M_d(\sigma) \in \mathbb{R}^{n\times n}$, $N_A(\sigma) \in \mathbb{R}^{n\times n}$, $N_B(\sigma) \in \mathbb{R}^{m\times m}$, and $N_d(\sigma) \in \mathbb{R}^{n\times n}$ are constant mode-dependent matrices and $F_A \in \mathbb{R}^{n\times n}$, $F_B \in \mathbb{R}^{m\times m}$, and $F_d \in \mathbb{R}^{n\times n}$ are unknown matrices that satisfy the inequalities $F_A^{'}(k)F_A(k) \leq I$, $F_B^{'}(k)F_B(k) \leq I$, $F_d^{'}(k)F_d(k) \leq I$ for all k.

Before tackling the stability problem of this class of systems, let us recall the following lemmas.

Lemma 1 *[7] Suppose that $W > 0$, $F'(k)F(k) \leq I$, and M, N are constant matrices of appropriate dimensions. If there exist a constant $\epsilon > 0$ such that $\epsilon I - M'WM > 0$ then,*

$$(A + MF(k)N)'\,W\,(A + MF(k)N) \leq \\ A'\big(W^{-1} - \epsilon^{-1}MM'\big)^{-1}A + \epsilon N'N.$$

Furthermore, if there exist $\epsilon > 0$ such that $\epsilon I - NWN' > 0$ then,

$$(A + MF(k)N)\,W\,(A + MF(k)N)' \leq \\ A\Big[W + WN'(\epsilon I - NWN')^{-1}NW\Big]A' + \epsilon MM'.$$

Lemma 2 *[15] For given matrices Σ_1, and Σ_2, we have*

$$\Sigma_1^{'}\Sigma_2 + \Sigma_2^{'}\Sigma_1 \leq \mu\Sigma_1^{'}\Sigma_1 + \mu^{-1}\Sigma_2^{'}\Sigma_2.$$

where μ is any positive constant.

Consider system (11) with $u_k \equiv 0$. If we take

$$V_k = x_k^{'}X^{-1}x_k + \sum_{i=k-d}^{k-1} x_i^{'}Z^{-1}x_i$$

as a common Lyapunov function associated to the dynamics (11) then, by exploiting the proof of theorem 1, we deduce from inequalities (6), and (9) the new stability conditions for the switched uncontrolled system (11). That is

$$-Z^{-1} + \Big(A_d(j) + \Delta A_d(k,j)\Big)'X^{-1} \\ \Big(A_d(j) + \Delta A_d(k,j)\Big) < 0, \quad \forall j \in \mathscr{S}, \tag{13}$$

and

$$\begin{aligned} &-X^{-1} + Z^{-1} - \Big(A(j) + \Delta A(k,j)\Big)' \times \\ &\times \Big(-X + \Big(A_d(j) + \Delta A_d(k,j)\Big)Z \\ &\Big(A_d(j) + \Delta A_d(k,j)\Big)'(\sigma)\Big)^{-1} \times \\ &\Big(A(j) + \Delta A(k,j)\Big) < 0, \quad \forall j \in \mathscr{S}. \end{aligned} \tag{14}$$

By the Schur complement lemma, condition (13) can be rewritten as

$$\begin{bmatrix} -Z^{-1} & A_d^{'}(j) \\ A_d(j) & -X \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ M_d(j) \end{bmatrix} F_d(k) \begin{bmatrix} N_d(j) & \mathbf{0} \end{bmatrix} \\ + \begin{bmatrix} N_d(j) & \mathbf{0} \end{bmatrix}' F_d^{'}(k) \begin{bmatrix} \mathbf{0} \\ M_d(j) \end{bmatrix}' < 0.$$

Using lemma 2, the left-hand side of the last inequality verifies

$$\begin{bmatrix} -Z^{-1} + \mu_j^{-1}N_d^{'}(j)N_d(j) & A_d^{'}(j) \\ A_d(j) & -X + \mu_j M_d(j)M_d^{'}(j) \end{bmatrix} \\ < 0; \ \forall \mu_j > 0.$$

Pre- and post multiplying the last inequality by the matrix $\begin{bmatrix} Z & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix}$ then, we obtain

$$\begin{bmatrix} -Z + \mu_j^{-1} Z\, N_d'(j) N_d(j) Z & Z\, A_d'(j) \\ A_d(j) Z & -X + \mu_j M_d(j) M_d'(j) \end{bmatrix} < 0 \tag{15}$$

which is equivalent by the Schur complement lemma to the following inequality

$$\begin{bmatrix} -Z & Z\, N_d'(j) & Z\, A_d'(j) \\ N_d(j) Z & -\mu_j I & \mathbf{0} \\ A_d(j) Z & \mathbf{0} & -X + \mu_j M_d(j) M_d'(j) \end{bmatrix} < 0. \tag{16}$$

Similarly, from inequality (14), and by the use of the Schur complement lemma, we write

$$\begin{aligned} &\begin{bmatrix} -X^{-1} + Z^{-1} & A'(j) \\ A(j) & -X \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{0} \\ M_A(j) \end{bmatrix} F_A(k) \begin{bmatrix} N_A(j) & \mathbf{0} \end{bmatrix} \\ &+ \begin{bmatrix} N_A(j) & \mathbf{0} \end{bmatrix}' F_A'(k) \begin{bmatrix} \mathbf{0} \\ M_A(j) \end{bmatrix}' \\ &+ \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (A_d(j) + \Delta A_d(k,j))\, Z\, (A_d(j) + \Delta A_d(k,j))' \end{bmatrix} \\ &< 0 \end{aligned}$$

Using lemma 2, we have

$$\begin{aligned} &\begin{bmatrix} -X^{-1} + Z^{-1} + \rho_j^{-1} N_A'(j) N_A(j) & A'(j) \\ A(j) & -X + \rho_j M_A(j) M_A'(j) \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (A_d(j) + \Delta A_d(k,j))\, Z\, (A_d(j) + \Delta A_d(k,j))' \end{bmatrix} \\ &< 0;\ \forall \rho_j > 0. \end{aligned}$$

By the use of lemma 1, and under the existence of a set of positive constants $(\epsilon_j)_{1\le j\le s}$ such that the following inequalities holds

$$\epsilon_j I - N_d(j) Z\, N_d'(j) > 0, \ \ \forall j \in \mathscr{S}, \tag{17}$$

Then,

$$\begin{aligned} &\Big(A_d(j) + \Delta A_d(k,j)\Big) Z \Big(A_d(j) + \Delta A_d(k,j)\Big)' \le \\ &A_d(j) \Big[Z + Z\, N_d'(j) \Big(\epsilon_j I - N_d(j) Z\, N_d'(j)\Big)^{-1} \\ &N_d(j) Z \Big] A_d'(j) + \epsilon_j M_d(j) M_d'(j) \\ &= \mathscr{K}(j). \end{aligned}$$

We conclude that if the following inequalities hold

$$\begin{bmatrix} -X^{-1} + Z^{-1} + \rho^{-1} N_A'(j) N_A(j) & A'(j) \\ A(j) & -X + \rho M_A(j) M_A'(j) + \mathscr{K}(j) \end{bmatrix} < 0;\ \forall j$$

then, the condition of stability (14) holds. Pre- and post multiplying the last inequality by the matrix $\begin{bmatrix} X & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix}$ then, we obtain for $1 \le j \le s$

$$\begin{bmatrix} -X + X\, Z^{-1} X + \rho_j^{-1} X_j N_A'(j) N_A(j) X & X\, A'(j) \\ A(j) X & -X + \rho_j M_A(j) M_A'(j) + \mathscr{K}(j) \end{bmatrix} < 0.$$

Applying the Schur complement lemma to the blocks $(1,1)$ and $(2,2)$ of the last matrix inequality we get for $1 \le j \le s$

$$\begin{bmatrix} -X & X & X N_A'(j) & X A'(j) & \mathbf{0} \\ X & -Z & \mathbf{0} & & \mathbf{0} \\ N_A(j) X & \mathbf{0} & -\rho_j I & \mathbf{0} & \mathbf{0} \\ A(j) X & \mathbf{0} & \mathbf{0} & & A_d(j) Z N_d'(j) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & N_d(j) Z A_d'(j) & -\epsilon_j I + N_d(j) Z N_d'(j) \end{bmatrix} < 0, \tag{18}$$

where $\mathscr{L}_{3,3}(j) = X - \rho_j M_A(j) M_A'(j) - A_d(j) Z A_d'(j) - \epsilon_j M_d(j) M_d'(j)$. We have proved the following statement.

Theorem 2 *Consider system (11) with $u_k \equiv 0$. If there exist sets of positive constants $(\epsilon_j)_{1\le j\le s}$, $(\mu_j)_{1\le j\le s}$, $(\rho_j)_{1\le j\le s}$, and two symmetric and positive definite matrices X and Z such that inequalities (16), (17), and (18) hold for $1 \le j \le s$ then, the uncontrolled system (11) is asymptotically stable under the presence of uncertainties $\Delta A(k,\sigma)$ and $\Delta A_d(k,\sigma)$.*

3. STABILIZABILITY

3.1. System without uncertainties

In this section we exploit the result of theorem 1 to analyze the stabilizability of system (1) by means of switching controllers. The design of the feedback gains is summarized in the following theorem.

Theorem 3 *Consider system (1). If there exist two symmetric and positive definite matrices $X \in \mathbb{R}^{n\times n}$, and Z, and a set of matrices $(Y_j)_{1\le j\le s} \in \mathbb{R}^{m\times n}$ such that for all $j \in \mathscr{S}$, we* *have*

$$\left[\begin{array}{ccc} -X & X & XA'(j)+Y_j^{'}B'(j) \\ X & -Z & \mathbf{0} \\ A(j)X+B(j)Y_j & \mathbf{0} & -X+A_d(j)ZA_d^{'}(j) \end{array}\right] < 0,$$
$$\left[\begin{array}{cc} -Z & ZA_d^{'}(j) \\ A_d(j)Z & -X \end{array}\right] < 0 \tag{19}$$

then, system (1) is stabilizable by the switching feedbacks

$$u_{k,j} = Y_j X^{-1} x_k. \tag{20}$$

Proof. By replacing $A(j)$ that appears in inequalities of theorem 1 by $A(j)+B(j)Y_jX^{-1}$, we obtain (19), which is the claim.

Remark 1 *A constant-gain stabilizing controller can be found by solving LMIs (19) with respect to a unique Y. Under the hypothesis of the existence of X, Y, and Z, the stabilizing controller $u_k = Y\,X^{-1}x_k$ achieves the asymptotic stability of system (1) without any care of both switching modes and switching instants.*

It is worthwhile to outline that the use of a common Lyapunov function does not prevent the search of switching feedbacks that depend on the system modes. The mode-dependent gains of feedback controllers are essentially given by the matrices $(Y_j)_{1\le j\le s}$ that are completely independent from the matrices X and Z. This technique can serve as an intelligent way to append the performances of switching controllers with a common Lyapunov function candidate.

3.2. System with uncertainties

Consider system (11). If we define

$$\xi_k = \left[\begin{array}{c} x_k \\ u_k \end{array}\right],\ \mathscr{A}(\sigma) = \left[\begin{array}{cc} A(\sigma) & B(\sigma) \\ \mathbf{0} & \mathbf{0} \end{array}\right],$$
$$\mathscr{A}_d(\sigma) = \left[\begin{array}{cc} A_d(\sigma) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right],\ \mathscr{B} = \left[\begin{array}{c} \mathbf{0} \\ I \end{array}\right],$$
$$\Delta\mathscr{A}(k,\sigma) = \left[\begin{array}{cc} \Delta A(k,\sigma) & \Delta B(k,\sigma) \\ \mathbf{0} & \mathbf{0} \end{array}\right],$$
$$\Delta\mathscr{A}_d(k,\sigma) = \left[\begin{array}{cc} \Delta A_d(k,\sigma) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right].$$

Then, system (11) is rewritten as

$$\begin{aligned} \xi_{k+1} &= \left(\mathscr{A}(\sigma)+\Delta\mathscr{A}(k,\sigma)\right)\xi_k \\ &\quad + \left(\mathscr{A}_d(\sigma)+\Delta\mathscr{A}_d(k,\sigma)\right)\xi_{k-d} + \mathscr{B}v_k, \end{aligned} \tag{21}$$

where $v_k = u_{k+1}$ is the new control input and

$$\begin{cases} \Delta\mathscr{A}(k,\sigma) = \mathscr{M}_A(\sigma)\mathscr{F}_A(k)\mathscr{N}_A(\sigma), \\ \Delta\mathscr{A}_d(k,\sigma) = \mathscr{M}_d(\sigma)\mathscr{F}_d(k)\mathscr{N}_d(\sigma), \end{cases} \tag{22}$$

such that

$$\mathscr{M}_A(\sigma) = \left[\begin{array}{cc} M_A(\sigma) & M_B(\sigma) \\ \mathbf{0} & \mathbf{0} \end{array}\right] \in \mathbb{R}^{(n+m)\times(n+m)},$$
$$\mathscr{F}_A(k,\sigma) = \left[\begin{array}{cc} F_A(k,\sigma) & \mathbf{0} \\ \mathbf{0} & F_B(k,\sigma) \end{array}\right] \in \mathbb{R}^{(n+m)\times(n+m)},$$
$$\mathscr{N}_A(\sigma) = \left[\begin{array}{cc} N_A(\sigma) & \mathbf{0} \\ \mathbf{0} & N_B(\sigma) \end{array}\right] \in \mathbb{R}^{(n+m)\times(n+m)},$$
$$\mathscr{M}_d(\sigma) = \left[\begin{array}{cc} M_d(\sigma) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \in \mathbb{R}^{(n+m)\times(n+m)},$$
$$\mathscr{F}_d(k,\sigma) = \left[\begin{array}{cc} F_d(k,\sigma) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \in \mathbb{R}^{(n+m)\times(n+m)},$$
$$\mathscr{N}_d(\sigma) = \left[\begin{array}{cc} N_d(\sigma) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \in \mathbb{R}^{(n+m)\times(n+m)}.$$

The stability of system (21) by a stabilizing controller of the form $v_k = K\xi_k$, where K is some design matrix, turns on the stability of the system

$$\begin{aligned} \xi_{k+1} &= \Big(\mathscr{A}(\sigma)+\Delta\mathscr{A}(k,\sigma)+\mathscr{B}K\Big)\xi_k \\ &\quad + \Big(\mathscr{A}_d(\sigma)+\Delta\mathscr{A}_d(k,\sigma)\Big)\xi_{k-d}, \end{aligned} \tag{23}$$

which is already analyzed in subsection 2.2. The design of the stabilizing switching feedbacks for system (21) is given in the following statement.

Theorem 4 *Consider system (21). If there exist two symmetric and positive definite matrices $\mathscr{X} \in \mathbb{R}^{(n+m)\times(n+m)}$, $\mathscr{Z} \in \mathbb{R}^{(n+m)\times(n+m)}$, a set of matrices $(\mathscr{Y}_j)_{1\le j\le s}$ of appropriate dimensions, and sets of positive constants $(\epsilon_j)_{1\le j\le s}$, $(\mu_j)_{1\le j\le s}$ and $(\rho_j)_{1\le j\le s}$ such that the following holds for $1\le j\le s$*

$$\epsilon_j I - \mathscr{N}_d(j)\mathscr{Z}\mathscr{N}_d^{'}(j) > 0,$$

$$\left[\begin{array}{ccc} -\mathscr{Z} & \mathscr{Z}\mathscr{N}_d^{'}(j) & \mathscr{Z}\mathscr{A}_d^{'}(j) \\ \mathscr{N}_d(j)\mathscr{Z} & -\mu_j I & \mathbf{0} \\ \mathscr{A}_d(j)\mathscr{Z} & \mathbf{0} & -\mathscr{X}+\mu_j\mathscr{M}_d(j)\mathscr{M}_d^{'}(j) \end{array}\right] < 0,$$

and

$$\left[\begin{array}{ccccc} -\mathscr{X} & \mathscr{X}_j & \mathscr{X}\mathscr{N}_A^{'}(j) & \mathscr{X}\mathscr{A}'(j)+\mathscr{Y}_j^{'}\mathscr{B}' & \mathbf{0} \\ \mathscr{X} & -\mathscr{Z} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathscr{N}_A(j)\mathscr{X} & \mathbf{0} & -\rho_j I & \mathbf{0} & \mathbf{0} \\ \mathscr{A}(j)\mathscr{X}+\mathscr{B}\mathscr{Y}_j & \mathbf{0} & \mathbf{0} & -\mathscr{L}_{3,3}(j) & \mathscr{A}_d(j)\mathscr{Z}\mathscr{N}_d^{'}(j) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathscr{N}_d(j)\mathscr{Z}\mathscr{A}_d^{'}(j) & -\epsilon_j I+\mathscr{N}_d(j)\mathscr{Z}\mathscr{N}_d^{'}(j) \end{array}\right] < 0,$$

where $\mathscr{L}_{3,3}(j) = \mathscr{X} - \rho_j\mathscr{M}_A(j)\mathscr{M}_A^{'}(j) - \mathscr{A}_d(j)\mathscr{Z}\mathscr{A}_d^{'}(j) - \epsilon_j\mathscr{M}_d(j)\mathscr{M}_d^{'}(j)$ then, system (21) is asymptotically stable under the actions of the switching feedbacks $v_{k,j} = \mathscr{Y}_j\,\mathscr{X}^{-1}\xi_k$.

Proof. From result of theorem 3, if we replace in Eqs. (16, 17, 18) the nominal matrix $A(j)$ by the closed-loop matrix $\mathscr{A}(j)+\mathscr{B}\mathscr{Y}_j\mathscr{X}^{-1}$, $A_d(j)$ by $\mathscr{A}_d(j)$, $N_A(j)$, by $\mathscr{N}_A(j)$, $M_A(j)$, by $\mathscr{M}_A(j)$, X_j by $\mathscr{X}$, and Z by $\mathscr{Z}$, we obtain the new LMIs conditions of theorem 4.

4. CONCLUSION

Sufficient delay-independent LMIs conditions for robust stability and stabilizability of switched discrete-time systems with delays are given. The formulation of the stabilizability issue in term of LMIs favorites also the setup of multi-objectives control design as H_∞ control or robust poles placement. This problem is not discussed in the current paper and will be investigated in our future works.

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